University of Manitoba

PUBLISHED BY THE DEPARTMENT OF MATHEMATICS

MANITOBA MATH LINKS

Beauty · Discovery · Principles

Volume 5, Number 11, Fall 2004

Sisler High School visits

Professor Lui (above left) is one of seven professors who accepted an invitation from Ken Gordon at Sisler High School to give a presentation. The other six presenters were Professors Mendelsohn, Craigen, Woods, Kucera, Penner, and Berry. The students (from left to right) are Anton Bokhanchuk, Rick Chan, Jolan Geronimo, Krizsia Praznik, Benedict Dacanay, Andrea Bellhouse, Alvin Agpalza, Christine Parto, Sean Restall, Daniel Truong, Ivy Mendiola, Amanda Cnudde, Ken Gordon, Olivia Macdonald-Mager. For the full story, see page 3. (Photo courtesy of Ken Gordon)

Manitoba Math Links is a newsletter published three times a year (Fall, Winter, Spring) by the Outreach Committee of the Department of Mathematics, University of Manitoba. *Manitoba Math Links* gratefully acknowledges past financial support from *The Winnipeg Foundation* and assistance from the Manitoba Department of Education and Youth.

Submissions

Manitoba Math Links welcomes material on any topic related to mathematics, including articles, applications, announcements, humour, anecdotes, problems, and history. Materials are subject to editorial revision. Submissions may be made by regular mail or electronically to either the general address, or directly to any of the editors.

We also welcome editorial comments or suggestions from students, teachers, parents, or anyone interested in mathematics and mathematics education. The format of this newsletter is constantly evolving, so any input is appreciated.

Need more copies? Please feel free to reproduce and distribute this newsletter as needed; electronic copies (.pdf) may also be downloaded from our website, which is on the home page (see below) of the Department of Mathematics.

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SPEAKERS AVAILABLE

If you are interested in having a faculty member come to your school and speak to students, please contact the Department of Mathematics or any member of the editorial board.

Contents

> As for everything else, so for a Mathematical theory: beauty can be perceived but not explained.
>
> —*Arthur Cayley (1821–1894)*

Announcement: The 11th Manitoba Workshop on Problem Solving for students in Senior 2,3 and 4 is now being planned for four Saturdays in mid to late January and early February 2005. For more information, visit the IIMS site `www.umanitoba.ca/institutes/iims/`, or contact Dr. Robert McLeod, Workshop coordinator (474-6724). We welcome Prof. Shaun Lui as the new mathematical content coordinator this year.

Sisler High School visits

Ken Gordon
Sisler High School

From May 14th to June 16th, 2004, 7 professors from the University on Manitoba's Department of Mathematics gave lectures on various enrichment topics to a class at Sisler High School. The students were from Sisler's University of Manitoba Calculus challenge program. They challenged both the Calculus I and Calculus II courses and spent the last two months of the course working on various enrichment materials. The presentations were part of this enrichment.

The idea for this lecture series was developed in February when I met Tom Berry at a workshop put on by the Department of Mathematics for math teachers in Winnipeg School Division #1. He indicated that there were professors interested in doing guest lectures at high schools, and it seemed like a perfect fit for the enrichment courses at Sisler.

Shaun Lui was the first presenter. Professor Lui's presentation consisted of outlining the major streams of pure math and focused in on studies in applied math. He gave some examples in acoustics, elasticity, fluid mechanics, and image processing. His presentation gave the students a big picture of what they could expect to study if they chose the field of applied mathematics.

Nathan Mendelsohn awed the students with his years of experience as he dealt with combinatorics. They quickly learned that there is more to the area than just permutations and combinations. He delved into the areas of networks and angles of polygons and also looked at Sperner's theorem and binary triples and quadruples.

Robert Craigen's topic of binary numbers was next as he gave two presentations on the subject. We learned all about Russian peasant multiplication, binary number arithmetic and how the two were linked together. He ended his first lecture with a look at how computers use binary numbers, which is where the second lecture began. The history of computers from the Jacquard loom to Moore's law was discussed. The second session ended with a look at a number of games that use binary numbers. NIM, the Towers of Hanoi, Chinese rings, and binary sort cards were all explored. For many of the students who had also studied computer sciences with me, it was a wonderful way to tie their computer studies to their math studies.

Grant Woods explored the concept of Pi and what happens to it when circles approach infinite size. While the concepts of dealing with spheres of near infinite size was a bit mind-boggling for the students, once they warmed to the idea they were entranced by the consequences. Dr. Woods' discussion opened some eyes to the danger of making assumptions in mathematics and how important it is to have a solid hypothesis.

Tom Kucera's lecture on mathematical logic left the students with their heads swimming with ideas. After he got through the initial introduction of the idea of Logic and it's terminology, his logic problems involving knights and knaves had the students putting their brains' processing functions into full gear. He also made some very astute observations about the importance of proofs in mathematics and the logic behind them. When his discussion ended with a few minutes remaining in the period, the class cried out for more logic problems to solve.

Peter Penner took the students to the Australian outback and the South Seas island of Vanuatu to explore mathematical anthropology. Drawing on his personal experiences in Africa, Dr. Penner explained how certain cultures have unique ways of setting up family relationships to ensure the genetic integrity of their population. The Walpiri of Australia were introduced as a closed community that has a seemingly complex system of marriage groupings involving patricycles and matricycles. However, when the system is looked at in reference to linear algebra and group theory the kinship systems become clearer. The tribes of Malekula Island in Vanuatu were also looked at as they had a similar system. A wonderful presentation that showed the students you can find math just about anywhere.

Tom Berry was the final presenter of the series. He discussed the idea of where an observer should sit in a movie theatre in order to get the 'best view' of the movie. After reviewing the mean value theorem and some inverse trigonometric differentiation, we were ready to begin. He took the students on a path that lead them to the optimization of a complex calculation. In order to try and simplify things, he then looked at the problem from a geometrical perspective. By combining the two ideas together we were able to reach a solution. It was an interesting problem that all of the students could relate to and at the same time one that they had the ability to solve. The combining of algebra and geometry to reach the solution emphasized to the class the importance of looking at mathematical problems from various perspectives.

All in all this has been a fantastic experience. It was invaluable for the students to meet and learn from members of the Department of Mathematics. Hopefully this will inspire them to further their studies in the area of mathematics as they begin their post secondary education. □

> One day, when I was doing well in class and had finished my lessons, I was sitting there trying to analyze the game of tic-tac-toe... The teacher came along and snatched my papers on which I had been doodling... She did not realize that analyzing tic-tac-toe can lead into dozens of non-trivial mathematical questions.
>
> —*Martin Gardner*
> *Math. Intell.* **19** (4), (Fall 1997), p.40.

The following is an article that appeared in the newsletter of The Fields Institute, Sept. 2003, reproduced with kind permission.

Why Support Research in Mathematics?

Kenneth R. Davidson
Director, Fields Institute

As a mathematician, I am frequently asked to explain what mathematical research is. Despite the widespread applications of technology in our everyday lives, nobody ever sees an equation when checking the weather channel or turning on a cell phone. Even people who have studied mathematics in university while pursuing a degree in science or engineering may still look upon mathematics as a collection of computational techniques. These are, of course, an integral part of mathematics. But they represent the tip of the iceberg, while the ballast is provided by a huge body of understanding that remains hidden beneath the surface from non-practitioners. The reality is that mathematics provides the framework for understanding almost any complicated phenomenon. Some mathematicians work directly on these interactions with other fields, but many others pursue problems without any direct application in mind. It is often the latter pursuits which paradoxically may have the greatest impact in the long run.

Mathematics is everywhere. It is difficult to identify any item produced by modern industry where mathematics did not play a crucial role somewhere along the line. In manufacturing, advanced mathematical techniques are used to model and test products on computers, to visualize design, to control robots, and to optimize production techniques. The computer itself was conceived (and early working models built) by mathematicians. Interpretations of hidden structure are made possible by the measurement of reflected waves-from MRIs to earthquakes-to locate brain tumours, to find oil, and to detect planets around distant suns. All of this was developed from mathematics which had its roots three hundred years ago. Along the way, the study moved from a simple physical situation deep into mathematical abstraction and back out to physics again.

Most computer and cell phone equipment uses mathematical techniques for encryption, not just for security but also for data compression and for ensured accuracy of transmission. The world telephone network requires extremely sophisticated algorithms to route with speed and efficiency millions of signals simultaneously, without the user's awareness.

Statistics is an indispensable tool in business, science, and even public policy. Indeed, we are inundated with statistics daily in the news. How do we actually know that 46.7 per cent of Canadians support the federal Liberal party?-did they ask you? Biologists are trying to understand the human genome with thirty thousand genes and potentially nine billion pairs of protein interactions as a result. They produce huge quantities of data that are meaningless until analysed by statistical methods. The banking and insurance industries rely on mathematics and statistics to evaluate risk, to make actuarial assessments of mortality, and to maintain proper cash reserves.

Mathematics is a way of thinking. Mathematics has been called "the language of science." This is not just a matter of vocabulary. Most complicated physical phenomena follow rather simple principles, generally expressed in precise mathematical formulas. What mathematics does for science is not just shorthand-it is a very different way of looking at and thinking about the problem. Indeed the mathematization of a "real" problem generally involves abstracting out general structures. Any pesky details such as actual measurements may be replaced by arbitrary constants. In this way, whole families of related problems are considered simultaneously, with the original problem merely a single instance.

At this point, the mathematics can take on a life of its own. Mathematicians may attack the problem by making simplifying assumptions that alter the original problem. In solving this simpler problem, they may identify important properties that can be measured and which yield key information about the whole system. These ideas then feed back into the larger context. Predictions from both the behaviour in the simpler theoretical models and from computational experiments in the more complicated situations lead mathematicians to make conjectures about the general case. Some such conjectures are so incisive and their solutions have such profound implications that they become world-famous problems. And eventually someone solves them! Frequently, many years after the original problem made its way from the real world to the symbolic world of mathematics, the answer comes back and causes a revolution in thinking.

Mathematics is predictive. Most people understand mathematics as a tool for calculation. Its public side is a huge "bag of tricks" for determining something you need to know from certain measurements. For example, how much fuel is needed for a 747 airplane with two hundred passengers and twenty tons of cargo to fly from New York to London? But even more important is the fact that mathematical models predict that certain things must happen, not just quantitatively but also qualitatively. Such predictions can often indicate new phenomena that scientists can see once mathematics tells them where to look for it.

Equally important, but more subtle, mathematics can often tell you that something is impossible. When most people say that something is impossible, they mean that, in their experience, it cannot be done and, further, they cannot imagine how it might be accomplished. (Most modern technology was in that category not so long ago.) You are always left with the nagging feeling that perhaps you are just not smart enough. But a mathematician can often say with absolute certainty that something cannot be done, not just with today's resources, but even with unimagined advances. Better yet, one can often identify an obstruction-a quantifiable measurement that calculates when one can do something and when one cannot. For example, a torus (doughnut) cannot be smoothly deformed into a sphere.

Finally, in a perverse turn of the screw, sometimes in attempting to show that a certain property is an obstruction, you find out that it is not! Overcoming this obstacle in its simplest instance can often lead to a solution in great generality.

Mathematics is beautiful. Mathematics is an art as well as a science. Hidden in rather simple rules (axioms) for a class of mathematical objects are many amazing consequences. Deep connections between ideas from different areas often result in quite a simple picture of how things fit together. The complicated morass of isolated examples becomes understandable in the big picture through the lens of a new idea.

Most mathematicians have a strong aesthetic sense of what good mathematics is. We distinguish a clean elegant argument from a grungy brute force approach, and we distinguish deep insights from routine extensions of old ideas. Some difficult new ideas arrive fully polished, but usually it is a messier business. The first breakthrough is a rough diamond. Full understanding often comes later, as others analyse the new technique, simplify, generalize, and polish it. Certain aspects hidden in a clever but opaque argument may be exposed and clarified. Over time, some results prove to be of central importance while others fall by the wayside, correct but of little lasting interest. Eventually the synthesis ends up in a textbook and becomes part of the standard lore of practitioners as well as experts.

Mathematics is hard to explain. I heard a talk by Jay Ingram, moderator of the television science series @discovery.ca. He described how difficult it is to explain science to the layman because the context is not there. Most people do not know the vocabulary of science and are unable to recognize scientific phenomena in their everyday lives. Well, in mathematics, we try to get our context by making a connection to science!

People see advances in technology every day. Engineering and drug companies get most of the credit. Behind the scenes, you may envisage an engineer doing wind tunnel experiments or a biologist identifying the gene that controls breast cancer. But it is not easy to see how a mathematician who can classify "finite simple group" fits into the picture. Indeed, I would find it difficult just to explain what a "simple group" is to a non-mathematician, yet the classification of such groups is a major achievement of twentieth-century mathematics.

Why support mathematical research? Since technology is advancing rapidly, turning ideas from science into useful products, you might believe that the best use of resources is to pour funds into that transition. In fact, this last step is both the best funded and least scientific. Many entrepreneurs, from basement inventors to multinational companies, put a lot of effort into producing technology for profit. But while real problems are overcome at this level, they are usually of an incremental nature.

Fundamental advances in science often occur unexpectedly. Of course, scientists expect to make advances, and are generally driven by long-term goals of understanding. Every once in a while, however, scientists are in exactly the right place at the right time. Then their experience and knowledge enable them to recognize and develop the fortuitous insight. Trying to predict in advance which investigations will produce important applications is not only futile but also dangerous. Both scientists and policy makers have a consistently bad record of charting the important future areas of development. Funding the most promising application instead of the most promising scientist chooses good public relations over good science.

Mathematics is often remote from the big payoff of a saleable product. Increasingly, however, sophisticated ideas from mathematics-whether new or old-play a critical role in all of science and engineering. The theoretical advances of today feed the practical advances of tomorrow.

Moreover, mathematics is relatively cheap! There are no big labs, no cyclotrons, or gene sequencers, or x-ray spectrographs. Mathematicians travel with their ideas in their heads, with pencils, paper and of course, computers. Contrary to popular mythology, mathematicians are very sociable; they love to talk about their mathematics (to other mathematicians, of course). If you bring together a group of talented mathematicians interested in a common problem, a lot will happen. A number of international mathematics institutes such the Fields Institute in Toronto have been established specifically to facilitate this kind of interaction. We seek out opportunities in any area of mathematical science where there is promising activity.

Mathematics is a living breathing growing subject. It is the fuel for science and technology. At work for the most part behind the scenes, mathematics makes it possible to understand the world we live in. □

> Let us grant that the pursuit of Mathematics is a divine madness of the human spirit, a refuge from the goading urgency of contingent happenings.
>
> —*Alfred North Whitehead (1861–1947):*

Classic Puzzles

D. Gunderson
Department of Mathematics

Three matches are on a table. Without adding another, make 4 out of 3. You are not allowed to break the matches. (From B. Kordemsky, *The Moscow Puzzles*, Charles Scribner's Sons, NY, 1972.)

The column **Cool Websites** will return next issue.

> There is no more common error than to assume that, because prolonged and accurate mathematical calculations have been made, the application of the result to some fact of nature is absolutely certain.
>
> —*Alfred North Whitehead (1861–1947)*

Not All πs are Created Equal

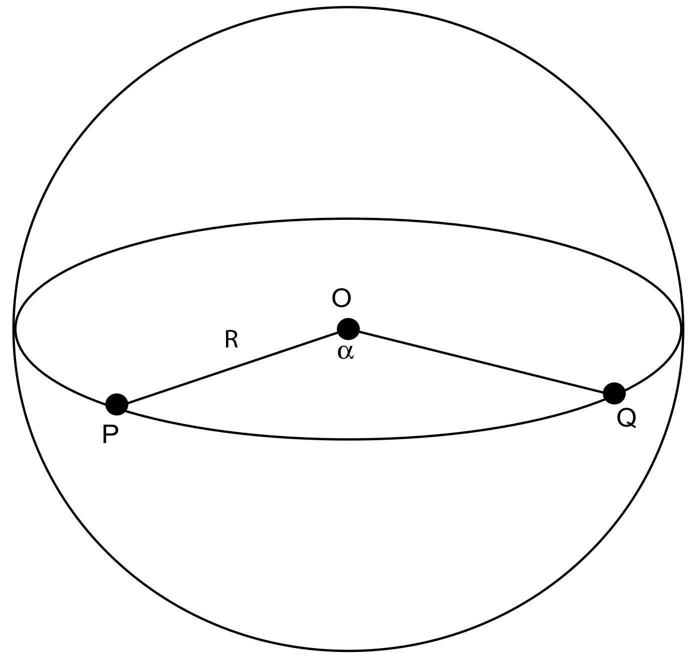
Grant Woods
Dept of Mathematics, U of M

Suppose you take a circle of diameter 10 cm, and divide its circumference by its diameter. What number do you get? Everybody "knows" that you get π, right? Now suppose that you take a bigger circle—say one of diameter 20 cm.— and again you divide its circumference by its diameter. What number do you get now? "You still get π", I hear you say. Well, suppose we had a really big circle, say one with a diameter of 10,000 light years, and we divided its circumference by its diameter. Is there a change in the answer? "Of course not" you reply. "For any circle at all, the ratio C/d of its circumference C to its diameter d (where both are measured using the same units of length) is π. Everybody knows that!

How do you know it? Why is it true that the ratio of the circumference to the diameter of a circle is the same for every circle, no matter how big or how small? "My Grade 6 teacher told me that, and she was smart and honest, so it must be true!" Not good enough! Of course, you could say "Nobody has ever found an example of a circle for which it is false." Not good enough—the absence of a counterexample is not the same as the presence of a proof. Is there a proof that it is true?

In this article I want to shake your confidence in this "fact" by describing a universe in which it is not true. This is a 2-dimensional universe whose 2-dimensional humanoids think that they live in an infinite flat universe, like a table-top extended to infinity in all directions. But they don't—they actually live on the surface of a sphere that is 300,000 light years in diameter. Their physicists have started investigating the geometry of very large circles, and they are discovering some disturbing things. For VERY big circles, the ratio C/d seems to be a bit smaller than π! Let's try to understand how this could happen.

First we must understand how we measure distances on the surface of a sphere. By "distance" we mean the shortest distance between two points on the sphere, remembering that our 2-dimensional humanoids must travel on the surface of the sphere—that's their universe. That distance is the length of the (shorter) arc of the "great circle" passing through them. If P and Q are two points in this universe, the great circle through them is the intersection of the universe (the surface of the sphere) with the unique plane that passes through P, Q, and the center O of the sphere. (Of course, O is not in the universe—the humanoids don't know about O.) The following sketch shows one choice for the points P and Q (and the great circle containing them).



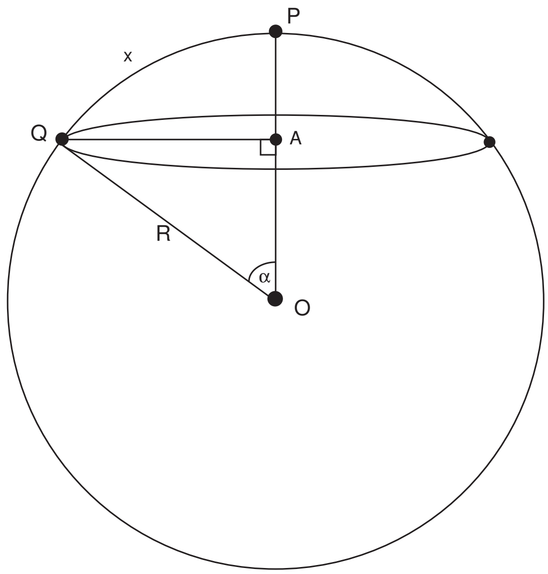
Suppose that the universe (*i.e.*, the surface of the sphere) has radius R. Suppose that $m\angle POQ = \alpha$, where α is measured in radians (recall that 360 degrees is 2π radians) and $\alpha < \pi$. Then the distance from P to Q is

$$\begin{aligned} &= \frac{\alpha}{2\pi} \times \text{circumference of the sphere} \\ &= \frac{\alpha}{2\pi} \times 2\pi R \\ &= \alpha R. \end{aligned}$$

(Of course, here we are assuming that our universe sits in "ordinary 3-dimensional space", where a circle of radius R does indeed always have circumference $2\pi R$. This may seem like circular reasoning, but it's only reasoning about circles.)

Now consider a circle in the humanoid universe centred at P and with radius x . As in our universe, in the humanoid universe this circle is defined to be the set of all points in the universe that are a distance x from P . These points form a circle (but not necessarily a great circle) on the surface of the sphere. For example, if the universe were the surface of the Earth, and P was at the North Pole, then the Arctic Circle would be a circle C^* centred at P in the humanoid universe.

Let's compute the circumference of this circle. First we need to find its centre (call it A) in the "real" 3-dimensional universe in which our sphere is embedded. Let Q be a point on our circle. It's not hard (I hope!) to see that A is the point on OP so that QA is perpendicular to OP (see the next figure).



Of course, A isn't in the humanoid universe either. As before, we assume that $m\angle POQ = \alpha$. Using basic trigonometry, the distance from A to Q is $R\sin\alpha$, and so the circumference of C^* is

$$2\pi \times (\text{“real” radius of } C^*) = 2\pi R\sin\alpha.$$

Thus, the ratio of the circumference of C^* to the diameter of C^* in the spherical universe is

$$\begin{aligned} &= \frac{2\pi R\sin\alpha}{2 \times \text{distance } PQ \text{ in the spherical universe}} \\ &= \frac{2\pi R\sin\alpha}{2R\alpha} = \pi\frac{\sin\alpha}{\alpha}. \end{aligned}$$

So the ratio of the circumference of C^* to its diameter is not a constant—it depends on the size of α! If α is very small then $\frac{\sin\alpha}{\alpha}$ is very close to 1 , so the ratio is almost exactly π. Any ordinary circle that would arise in any "practical" application that our humanoids would have would be like this. So they, like us, would conclude that the ratio of the circumference of a circle to its diameter is always π, no matter what the size of the circle is. Only after they developed the technology to measure accurately the radius and circumference of circles many light-years across would they discover that the ratio stopped being so close to π.

Here are some questions to contemplate.

1. Show that the circumference/diameter ratio gets smaller in the spherical universe as the radius of the circle gets larger.

2. Show that if a circle in the spherical universe has a radius that is 1/12 th of the circumference of the universe then $\pi = 3$ for that circle.

3. Is our 3-dimensional universe "flat" or is it "curved" like the humanoid spherical universe? (Cosmologists wonder about this.) Do all our circles have the same circumference/diameter ratio? What geometric axiom will guarantee that they do? Does our universe satisfy that axiom?

And... perhaps you shouldn't believe what your Grade 6 teacher (or anyone else) tells you without first thinking it over for yourself. □

Math Camp 2004

The students

Math Camp 2004 was not what we expected. It was not full of nerdy geeks who know the first thirty digits of π or the 70th term of the Fibonacci sequence. Instead, it was filled with normal kids who are simply good at math, and who like being good at math. We all came for different reasons, from making new friends to learning and understanding new math concepts, and we all came away with both.

One of the great things about math camp was that all of us twenty students were qualified to be here. We all understand simple terms and had enough background that the lectures were interesting, and full of new information that was challenging. While working in groups, the other students could help you with problems, because even if you didn't understand something, someone else did, not that they needed to help you, because the teachers were overly willing to help. There was Bill, our father for the week, who was always giving us new problems and distracting us from doing our questions, and Will, who, as one camper said, "is da BOMB". Oh, and Manon, who was just like one of us kids, searched the university to find some games of *SET*, just because we wanted to play. Most of all, there was Dr. Trim, who gets so into his lessons, it make everyone else get so into them, too.

During the five days, we had two guest speakers from the Department of Mathematics here at the University of Manitoba. They spoke about concepts including the pigeonhole principle, Ramsey theory, and a discussion on "what is mathematics" to which we came up with, "FUN".

Math Camp was an unforgettable experience. Hanging out at the University and staying in the dorms made us feel like we were university students, and gave us a feeling of the perks of coming to camp. The food was amazing, we got to play sports and games, a great field trip to *DQ*, and best of all, Math Camp T-shirts.

And finally, the best part, for each and everyone of us, was the people. Hanging out with nineteen other teens your age for five days can only result in good friends, great times, and even better memories. □

> Examinations are formidable even to the best prepared, for the greatest fool may ask more than the wisest man can answer.
>
> —*Charles Caleb Colton (1825)*

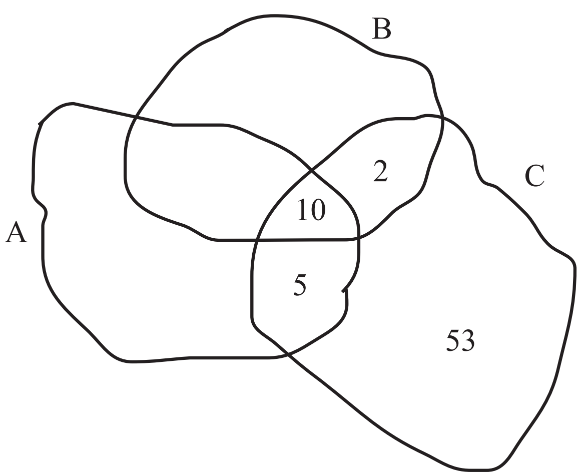
PROBLEM CORNER

D. Trim
Department of Mathematics

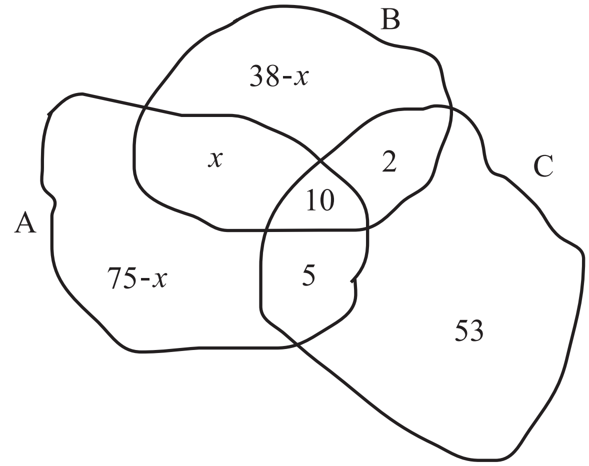
Dear Readers:

Welcome once again to the PROBLEM CORNER. Here is the problem from the last column and its solution: A group of 150 students are enrolled in at least one of the three first-year mathematics classes: A, B, and C. There are 90 students in class A, 50 in class B, 70 in class C, 15 in classes A and C, 12 in classes B and C, and 10 in all three classes. Determine how many students are enrolled in both classes A and B.

In problems of this type, it is advantageous to make what is called a Venn diagram showing all the given information. The first step is to draw overlapping regions representing A, B, and C. Next we place a 10 in the region common to A, B, and C since 10 students take all three classes. Because 15 students are enrolled in A and C, we place the 5 as shown. Similarly, the 2 results from the fact that 12 students are taking B and C. We can now place the 53 since 70 students take C.



To add numbers to the remaining three regions, we let the number of students taking A and B, but not C, be x. It is then possible to complete the diagram as shown below using the facts that 90 students are in class A and 50 in class B.



Since the total number of students is 150, we can write the equation

$$150 = (38 - x) + x + 10 + 2 + (75 - x) + 5 + 53.$$

The solution of this equation is $x = 33$. The number of students taking courses A and B is therefore $33 + 10 = 43$.

I am delighted to report that we had six submissions to this problem, five individual submissions from Westwood Collegiate, and a submission from the Grade 11 class at Elm Creek School. Of the Westwood submissions, Jeremy Comrie was closest; he obtained $x = 33$, and stated that 33 students are taking A and B only. This was a correct conclusion, but not the final answer. Duncan Pollett, David Ross, and Sebestian Cichosz all calculated $x = 33$ as well, and gave this as the number of students in A and B. Elm Creek had the correct procedure, as outlined above, but slipped up in formulating the equation.

Congratulations to everyone for their submissions. I hope that we will see more in the future. Send submissions on the next problem to:

S. Kangas,
Department of Mathematics,
The University of Manitoba,
Winnipeg, MB
R3T 2N2

Here is your new problem: Show that the second last digit of 2137^{753} cannot be even.