

Why did the chicken cross the Möbius strip? To get to the SAME side!

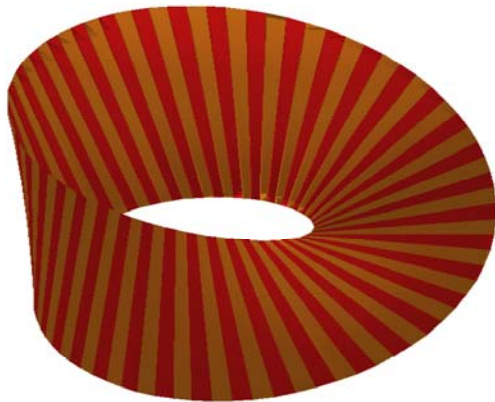
We get this twist on the classic “why did the chicken cross the road ? To get to the OTHER side!” due to the fact that the Möbius strip has, in fact, only one, *never-ending* side.

A Möbius strip (or band) can be obtained from a rectangular sheet by gluing two edges as in the following picture:



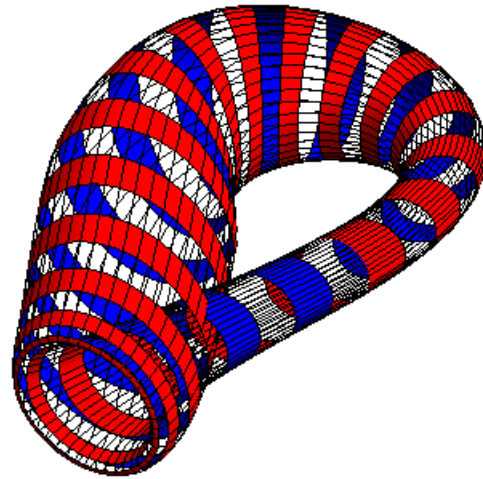
(Or, in other words, take a long strip of paper, give it one twist, then glue the edges together.)

The result is shown in the next figure (the figure on the right is M.C. Escher's playful use of the Möbius strip. Notice that these ants could walk and walk and walk forever without ever reaching the “end” of the strip):



This “surface”, along with many others, is the topic of study in the mathematical field known as “Topology”. Topology is a central mathematical field; it is a tool for imagining and studying various spaces or objects.

To take the Möbius strip to the next dimension, glue together the two short, almost-parallel sides of each red or orange rectangles on the Möbius band that we see above. The result is an exotic space called Klein bottle. The Klein bottle cannot be properly presented in three dimensions (we need at least four dimensions), and so the two illustrations below show some intersections that are not present.



You can study these surfaces and many others in MATH 3240, Introduction to Point-Set Topology.