MATH 1010 Midterm B November, 2007

Instructions:
1. Answer all questions on the machine-scored answer sheet provided. Use pencil only.
2. Return examination paper with machine-scored answer sheet.
4. Fill in the information requested below.
5. The examination invigilators may not interpret or explain questions to you.
6. Fill in your student number on the machine-scored sheet and encode it as well.

FAMILY NAME __________________________ FIRST NAME __________________________

STUDENT NUMBER __________________________

SIGNATURE __________________________

INSTRUCTOR __________________________

For the first seven problems, use the following matrices:

\[
A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 4 \\ 1 & 3 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 \\ 3 & 2 \\ 2 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 3 \\ -1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 3 & 3 \\ -1 & 4 & 5 \end{bmatrix}.
\]

1. What is the (3, 2) entry of \( AB \)?
   (a) -8 (b) 37 (c) -18 (d) 33 (e) None of these

2. What is the (3, 1) entry of \( AD^T \)?
   (a) 25 (b) 20 (c) 13 (d) Does not exist (e) None of these

3. What is the (2, 2) entry of \( D - 2B^T \)?
   (a) 8 (b) 0 (c) -4 (d) Does not exist (e) None of these

4. What is the (3, 3) entry of \( BC \)?
   (a) -12 (b) 3 (c) -4 (d) Does not exist (e) None of these

5. What is the (2, 3) entry of \( A^2 \)?
   (a) 49 (b) 16 (c) 20 (d) 30 (e) None of these

6. What are the dimensions of the matrix \( ABC \)?
   (a) \( 2 \times 3 \) (b) \( 3 \times 2 \) (c) \( 1 \times 3 \) (d) \( 3 \times 1 \) (e) None of these

7. Which of the following are possible dimensions for a matrix \( E \) if the product \( BE \) is defined?
   (a) \( 2 \times 3 \) (b) \( 3 \times 5 \) (c) \( 3 \times 2 \) (d) \( 1 \times 3 \) (e) None of these

8. What is the (2, 1) entry of a 3 \( \times \) 3 matrix whose entries are defined by the equation \( a_{ij} = i + 2j \)?
   (a) 1 (b) 2 (c) 0 (d) 4 (e) None of these

9. The accompanying matrix is not in row echelon form.
   What is the reason?
   (a) not enough leading 1's
   (b) not enough 0's above leading 1's
   (c) Not enough 0's below leading 1's
   (d) not all leading 1's in correct places
   (e) the matrix is actually in row echelon form

\[
\begin{bmatrix}
1 & 2 & 3 & 2 & 3 & 6 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -3 & -2
\end{bmatrix}
\]
10. What single elementary row operation would simplify the accompanying matrix to reduced row echelon form?
(a) interchange two rows
(b) interchange two columns
(c) add a multiple of one row to another row
(d) multiply a row by a nonzero number
(e) none of the above

11. When a row echelon form for a matrix is different than the reduced row echelon form for the same matrix, the difference is:
(a) the number of leading 1’s
(b) the number of rows of 0’s
(c) the positions of leading 1’s
(d) the number of 0’s in columns with leading 1’s
(e) none of the above

12. You are given that the reduced row echelon form for the augmented matrix associated with the system of equations
\[ \begin{align*}
    x - 2y - 4z &= 11 \\
    x - y - 2z &= 7 \\
    3x - 4y - 8z &= 25
\end{align*} \]

is \[
\begin{pmatrix}
    1 & 0 & 0 & 3 \\
    0 & 1 & 2 & -4 \\
    0 & 0 & 0 & 0
\end{pmatrix}
\]

The solution of the system is:
(a) \( x = 3, y = -4, z = 0 \)
(b) \( x = 3z, y = -2z - 4, \text{ where } z \text{ is arbitrary} \)
(c) \( x = 3, y = -2z - 4, \text{ where } z \text{ is arbitrary} \)
(d) the system does not have a solution
(e) none of the above

13. If \[
\begin{pmatrix}
    2 & -3 & 4 & 6 & 4 & 2 \\
    3 & 4 & -1 & 2 & 5 & -10 \\
    6 & 0 & 4 & 0 & 1 & 19
\end{pmatrix}
\]
is the augmented matrix for a system of linear equations, then:
(a) The system has 4 unknowns and 3 equations
(b) The system has 5 unknowns and 3 equations
(c) The system has 3 unknowns and 5 equations
(d) It is impossible to determine the number of equations and unknowns
(e) None of the above

14. Which of the following matrices is the reduced row echelon form for the augmented matrix to the right?
\[
\begin{pmatrix}
    1 & 0 & 2 & -3 & 5 \\
    0 & 1 & 1 & 2 & 1 \\
    0 & 0 & 2 & 4 & 6
\end{pmatrix}
\]

(a) \[
\begin{pmatrix}
    1 & 0 & 2 & -3 & 5 \\
    0 & 0 & 1 & 2 & 3 \\
    0 & 1 & 1 & 2 & 1
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
    1 & 0 & 0 & -7 & -1 \\
    0 & 1 & 0 & 0 & -2 \\
    0 & 0 & 1 & 2 & 3
\end{pmatrix}
\]
(c) \[
\begin{pmatrix}
    1 & 0 & 0 & 1 & 11 \\
    0 & 1 & 0 & 4 & 4 \\
    0 & 0 & 1 & 2 & 3
\end{pmatrix}
\]
(d) \[
\begin{pmatrix}
    1 & 0 & 0 & -7 & 3 \\
    0 & 1 & 1 & 2 & 1 \\
    0 & 0 & 0 & 0 & 4
\end{pmatrix}
\]
(e) None of the above
15. When the augmented matrix for a system of three equations in 4 unknowns is partially simplified, the matrix to the right is obtained. What do you conclude about the number of solutions of the equations?

- (a) There is exactly one solution.
- (b) There is no solution.
- (c) There is an infinity of solutions.
- (d) There is not yet enough information to conclude how many solutions the system has.
- (e) None of the above

\[
\begin{bmatrix}
1 & 6 & 2 & -3 & 5 \\
0 & 1 & 3 & 4 & 2 \\
0 & 0 & 3 & 4 & 1 \\
\end{bmatrix}
\]

16. If the matrix to the right is simplified to row echelon form, how many leading 1’s will it have?

- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) None of these

\[
\begin{bmatrix}
1 & 3 & 0 & 3 & 2 \\
0 & 0 & 1 & 2 & 3 \\
0 & 0 & 0 & 1 & 4 \\
0 & 0 & 0 & 2 & 9 \\
\end{bmatrix}
\]

17. Which entry in the augmented matrix \(
\begin{bmatrix}
1 & 2 & 0 & -1 & 2 \\
0 & 0 & 1 & 0 & -3 \\
0 & 0 & 0 & 1 & 4 \\
\end{bmatrix}
\) prevents it from being in REF?

- (a) the 2 in position (1,2)
- (b) the 0 in position (2,4)
- (c) the 4 in position (3,5)
- (d) something else
- (e) the matrix is in row echelon form

18. If the inverse of the matrix \( A = \begin{bmatrix} 1 & -1 & -1 \\ 2 & -3 & 4 \\ 3 & 1 & -2 \end{bmatrix} \) is \( A^{-1} = \frac{1}{25} \begin{bmatrix} -2 & 3 & 7 \\ -16 & -1 & 6 \\ -11 & 4 & 1 \end{bmatrix} \), then the solution of the system of equations

\[
\begin{align*}
x - y - z &= 1, \\
2x - 3y + 4z &= 3, \\
3x + y - 2z &= -1,
\end{align*}
\]

for \( y \) is:

- (a) 25
- (b) -25
- (c) -1
- (d) 1
- (e) None of these

19. Which of the following matrices is the inverse of \( A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ -2 & -3 & -4 \end{bmatrix} \)?

- (a) \( \begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & 3 \\ -2 & 4 & 0 \end{bmatrix} \)
- (b) \( \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \)
- (c) \( \begin{bmatrix} 4 & 1 & 2 \\ -4 & -3 & -3 \\ 1 & 1 & 1 \end{bmatrix} \)
- (d) \( \begin{bmatrix} 4 & 1 & 2 \\ -4 & -2 & -3 \\ 1 & 1 & 1 \end{bmatrix} \)
- (e) None of these
20. Which of the following four graphs is not simple?

(a) ![Graph A]
(b) ![Graph B]
(c) ![Graph C]
(d) ![Graph D]

(e) They are all simple.

21. If the degrees of the nodes of a graph are \(\{2, 4, 3, 3, 4, 2\}\), how many edges does it have?

(a) 36  (b) 6  (c) 9  (d) 18  (e) We cannot tell.

22. A graph is known to have 16 edges. If 5 nodes have degree 4 and all other nodes have degree 3, how many nodes does the graph have in total?

(a) 4  (b) 9  (c) 12  (d) 32  (e) None of these

23. Which of the following is not possible as the degrees for the nodes of a simple graph?

(a) \(\{2, 1, 3, 3\}\)  (b) \(\{0, 1, 3, 2, 2\}\)  (c) \(\{2, 2, 2, 2\}\)  (d) \(\{1, 1, 2, 2\}\)

(c) All of these are possible

24. The following graphs are

(a) not equivalent because they do not have the same number of nodes
(b) not equivalent because they do not have the same number of edges
(c) not equivalent because they do not have the same degree sets
(d) not equivalent for some other reason
(e) equivalent.

25. How many nonequivalent simple graphs can be drawn with 4 nodes, one of which must be isolated?

(a) 1  (b) 2  (c) 3  (d) 4  (e) More than 4