INSTRUCTIONS:
1. Single-line display calculators are permitted. No other aids.
2. Attempt all questions.
3. Answer the multiple choice questions in pencil on the bubble sheet provided.
4. Write solutions to long-answer problems 13–18 in the space provided.
5. Check that your examination booklet contains pages numbered from 1 to 9.
6. Fill in the information requested below.
7. Return all materials at the end of the examination.

Student Name (Print):  Family ___________  First ___________

Student Signature:  _______________________________________

Student Number:  _______________________________________

Instructor’s Name:  _______________________________________

(R. Borgersen for A01, D. Trim for A02, T. Mohammed for A03)

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Maximum Mark</th>
<th>Assigned Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–12</td>
<td>2 each</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Examination Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
1. What is the equation of the line passing through the point (1, -3) and having slope -4?
   (a) 4y - x = 7  (b) 4x + y = 7  (c) y - 4x + 7 = 0  (d) 4x + y = 1  (e) None of these

2. Which of the following augmented matrices is in reduced row echelon form?
   \[
   \begin{pmatrix}
   1 & 2 & 0 & | & 2 \\
   0 & 0 & 1 & | & -1 \\
   0 & 0 & 0 & | & 0
   \end{pmatrix}  \quad \quad \quad
   \begin{pmatrix}
   1 & 0 & 3 & | & 3 \\
   0 & 0 & 0 & | & 2 \\
   1 & 0 & 0 & | & -3
   \end{pmatrix}
   \]
   (a) only (i)  (b) only (ii)  (c) both (i) and (ii)  (d) neither (i) nor (ii)

3. You are told that the inverse of the matrix \( A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ -2 & 1 & 2 \end{pmatrix} \) is \( A^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & x \\ 1 & 0 & 1 \end{pmatrix} \).
   What is the value of \( x \)?
   (a) 3  (b) 0  (c) 2  (d) -1  (e) None of these

4. If the following matrix represents the augmented matrix of a system of linear equations, how many solutions does the system have?
   \[
   \begin{pmatrix}
   1 & 2 & 3 & | & 5 \\
   0 & 1 & 1 & | & 4 \\
   0 & 1 & 2 & | & 9 \\
   0 & 2 & 4 & | & 18
   \end{pmatrix}
   \]
   (a) one  (b) two  (c) none  (d) infinitely many  (e) None of these

5. If \( A = \begin{pmatrix} 2 & 1 \\ 3 & -1 \\ -2 & 5 \end{pmatrix} \) and \( B = \begin{pmatrix} 1 & 3 \\ 0 & -2 \\ 6 & 5 \end{pmatrix} \), what is the (2, 2) entry of \( 2AB^T - 3I \), where \( I \) is the 3x3 identity matrix?
   (a) -7  (b) 1  (c) 0  (d) 7  (e) None of these

6. Which entry in the augmented matrix \( \begin{pmatrix} 1 & 2 & 0 & -1 & 2 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 4 \end{pmatrix} \) prevents it from being in REF?
   (a) the 2 in position (1, 2)  (b) the 0 in position (2, 4)  
   (c) the 4 in position (3, 5)  (d) something else  (e) the matrix is in row echelon form

7. If the matrix to the right is simplified to row echelon form, how many leading 1's will it have?
   \[
   \begin{pmatrix}
   1 & 3 & 0 & 3 & 2 \\
   0 & 0 & 1 & 2 & 3 \\
   0 & 0 & 0 & 1 & 4 \\
   0 & 0 & 0 & 2 & 9
   \end{pmatrix}
   \]
   (a) 1  (b) 2  (c) 3  (d) 4  (e) None of these
8. You are given that the reduced row echelon form for the augmented matrix associated with the system of equations

\[
\begin{align*}
    x - 2y - 4z &= 11 \\
    x - y - 2z &= 7 \\
    3x - 4y - 8z &= 25 \\
\end{align*}
\]

is

\[
\begin{pmatrix}
    1 & 0 & 0 & | & 3 \\
    0 & 1 & 2 & | & -4 \\
    0 & 0 & 0 & | & 0 \\
\end{pmatrix}
\]

The solution of the system is:
(a) \(x = 3, y = -4, z = 0\)
(b) \(x = 3z, y = -2z - 4, \) where \(z\) is arbitrary
(c) \(x = 3, y = -2z - 4, \) where \(z\) is arbitrary
(d) the system does not have a solution
(e) none of the above

9. The following graphs are
(a) not equivalent because they do not have the same number of nodes
(b) not equivalent because they do not have the same number of edges
(c) not equivalent because they do not have the same degree sets
(d) not equivalent for some other reason
(e) equivalent.

10. A graph has 12 edges and 8 nodes. Four nodes are known to have degree 3, and two nodes have degree 4. If the remaining nodes all have the same degree, what is this degree?
(a) 1 (b) 2 (c) 3 (d) 4 (e) None of these

11. The adjacency matrix for a graph with 4 nodes is

\[
A = \begin{pmatrix}
    1 & 1 & 1 & 1 \\
    1 & 0 & 1 & 0 \\
    1 & 1 & 1 & 2 \\
    1 & 0 & 2 & 0 \\
\end{pmatrix}
\]

What is the degree of node 3?
(a) 2 (b) 4 (c) 6 (d) 8 (e) None of these

12. An investment of $500 receives simple interest at a rate of 4%. What is the total value of the investment including interest earned after 6 years and 6 months?
(a) $648.18 (b) $630.00 (c) $30.00 (d) None of these
16 13. A linear programming problem is to minimize the function $z = 3x + 2y$ subject to the constraints

$$x + y \geq 8,$$
$$5x + 12y \geq 60,$$
$$x \geq 0,$$
$$y \geq 0.$$

(a) Sketch the feasible set.
(b) Determine all corners for the feasible set.
(c) Find the optimal solution for the problem.
10 14. The Maple Leaf Company manufactures one-ton, two-ton, and four-ton trucks at two manufacturing plants, Plant A and Plant B. The number of trucks produced by each plant per week and weekly production costs for each plant are given in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Plant A</th>
<th>Plant B</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-ton trucks/week</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>Two-ton trucks/week</td>
<td>75</td>
<td>100</td>
</tr>
<tr>
<td>Four-ton trucks/week</td>
<td>50</td>
<td>150</td>
</tr>
<tr>
<td>Weekly cost</td>
<td>$4,000,000</td>
<td>$6,000,000</td>
</tr>
</tbody>
</table>

The company has an order to deliver 5000 one-ton trucks, 7500 two-ton trucks and 6000 four-ton trucks. Set up, **BUT DO NOT SOLVE**, a linear programming problem to determine the number of weeks each plant should operate in order to produce at least the required number of trucks at minimum cost. Your solution should contain the objective function and all constraints on all variables.
15. Consider the following graph.

(a) Is this graph simple? Explain why.

(b) Is this graph planar? (Circle yes or no) Yes No

(c) What is the degree set for the graph? Write the degrees in increasing order.

(d) Find an example of each of the following in the graph, or explain why they do not exist.

(i) An Euler circuit.

(ii) An Euler path that is not an Euler circuit.
10 16. Consider the following graph.

(a) If the graph has Hamiltonian circuits, find one. If it does not, explain why not.

(b) What is the adjacency matrix for the graph?

(c) Use the adjacency matrix to determine the number of routes of length 2 from node B to node A. Show you work.
You may use the following formulas for the remaining questions.

\[ S_n = P \left(1 + \frac{in}{100m}\right) \quad C_n = P \left(1 + \frac{i}{100m}\right)^n \quad A_n = P \left[\left(1 + \frac{i}{100m}\right)^n - 1\right] \frac{i}{100m} \]

11 17. (a) An investment of $P$ earned interest at the rate of 4% compounded quarterly. If the investment grew to $15,000 in $10\frac{1}{2}$ years, how much was $P$?

(b) Beginning May 1, 2009, you make 60 equal monthly payments of $100 into an account that earns 3% interest, compounded monthly. How much is in the account May 1, 2014?
18. (a) Calculate the monthly payments on a 20-year mortgage of $200,000 at 5% compounded monthly.

(b) How much remains to be paid on the mortgage in part (c) immediately after making the 60th payment?