THE UNIVERSITY OF MANITOBA

DATE: April 29, 2005
PAPER # 757
DEPARTMENT & COURSE NO: 136.101
EXAMINATION: Applied Finite

FINAL EXAMINATION
TITLE PAGE
TIME: 2 hours
EXAMINER: P. Penner, M. Davidson

LAST NAME: ____________________________
GIVEN NAME: __________________________
STUDENT NUMBER: ______________________
SEAT NUMBER: __________________________

SIGNATURE: (in ink) ____________________________
(I understand that cheating is a serious offense)

IMPORTANT: Please indicate your instructor and section by placing a check mark in
the appropriate box below.

☐ L04 Tue & Thurs. 10:00 – 11:30 am  P. Penner
☐ L05 M,W,F, 12:30 – 1:30 pm  M. Davidson

INSTRUCTIONS TO STUDENTS:

1. This is a 2 hour exam.

2. Answer all questions in Part A – Multiple Choice on the machine answer sheet
provided.

3. Answer all questions in Part B – in the spaces provided on this examination paper.
Show details of your work for full marks in Part B.

4. No calculators or cell phones permitted.

5. This exam has a title page, 7 pages of questions and 2 blank pages for rough work.
Please check that you have all the pages. You may remove the two blank pages from
this exam, but do so carefully without loosening the staple. DO NOT TEAR ANY
OTHER SHEETS OUT OF THIS EXAMINATION PAPER. You must hand back
the entire exam paper (except for the scrap paper).

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<th>Score</th>
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<td>Part B Question 2</td>
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<td>70</td>
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</table>
Part A – Multiple Choice (28 marks)

1. What is the y-intercept of the line $4y + 7x = 12$?
   a) 3  
   b) 0  
   c) $-1$  
   d) $\frac{12}{7}$  
   e) $\frac{-7}{4}$

2. What is the slope of the line $4y + 7x = 12$?
   a) $\frac{-7}{4}$  
   b) $\frac{12}{7}$  
   c) $\frac{4}{7}$  
   d) 3  
   e) 0

3. What is the x value of the point on the line $4y + 7x = 12$ having a y value of $-4$?
   a) 10  
   b) 4  
   c) $-4$  
   d) 0  
   e) 7

4. What is an equation of a line which passes through the point $(1, -1)$ and $(3, 5)$?
   a) $y + 3x = 4$  
   b) $y - 3x = -4$  
   c) $y + x = 0$  
   d) $2x - y = 1$  
   e) none of the above

5. What is an equation of a line which has a slope of 2 and passes through the point $(-3, 1)$?
   a) $y = 2x + 1$  
   b) $y = 2x - 3$  
   c) $4x - 2y = -14$  
   d) $3y + x = 0$  
   e) none of the above

6. What is an equation of a line that is perpendicular to the line from Question 5, and passes through the point $(4, 1)$?
   a) $4y + x = 0$  
   b) $2y = x + 1$  
   c) $y = 2x - 1$  
   d) $2y + x = -2$  
   e) none of the above

7. The inequality $2y + 3x + 4 \leq 3y - x + 8$ is equivalent to
   a) $\frac{2}{3}y \leq \frac{1}{3}x - 2$  
   b) $y \leq 4x + 4$  
   c) $y \geq 4x - 4$  
   d) $\frac{2}{3}y \geq -\frac{1}{3}x + 2$  
   e) none of the above

8. The inequality $x - 3y \geq 7$ is satisfied by
   a) $(7, 1)$  
   b) $(-2, -3)$  
   c) $(9, 2)$  
   d) $(1, -1)$  
   e) none of the above
For the next five questions, use the following matrices:

\[
A = \begin{bmatrix} 1 & 0 & 2 \\ 4 & 1 & -3 \\ -1 & 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 2 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ -1 & 3 \end{bmatrix}
\]

9. What is the (2,2) entry of AB?
   a) 1
   b) 2
   c) 3
   d) does not exist
   e) none of the above

10. Is BC = CB?
    a) yes
    b) no

11. What is the (1,1) entry of CA?
    a) 0
    b) -11
    c) 12
    d) 7
    e) does not exist

12. What is the (3,3) entry of BC?
    a) 1
    b) -2
    c) 0
    d) 3
    e) does not exist

13. What is the (1,2) entry of \( D^T A \)?
    a) -1
    b) 2
    c) -3
    d) 3
    e) none of the above

14. If the augmented matrix of a system of equations is
    \[
    A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ -1 & -1 & 3 & 2 \end{bmatrix},
    \]
    what two operations were performed to get the matrix
    \[
    B = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & -3 & -5 & 1 \end{bmatrix}
    \]
    a) \( R_2 \leftarrow R_2 + 2R_1 \); \( R_3 \leftarrow R_3 - R_1 \)
    b) \( R_3 \leftarrow R_3 + \frac{1}{2}R_2 \); \( R_2 \leftarrow R_2 + R_1 \)
    c) \( R_2 \leftarrow R_2 - 2R_1 \); \( R_3 \leftarrow R_3 + R_1 \)
    d) \( R_2 \leftarrow R_2 + 2R_3 \); \( R_3 \leftarrow R_3 + 2R_1 \)
    e) none of the above

15. What two operations were performed on \( B \) from Question 14 to get the matrix
    \[
    C = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}
    \]
    a) \( R_1 \leftarrow R_2 \); \( R_3 \leftarrow R_3 + \frac{1}{3}R_1 \)
    b) \( R_2 \leftarrow R_3 \); \( R_3 \leftarrow R_3 + 3R_2 \)
    c) \( R_2 \leftarrow -\frac{1}{3}R_2 \); \( R_3 \leftarrow R_3 - R_2 \)
    d) \( R_1 \leftarrow R_1 - 2R_3 \); \( R_2 \leftarrow R_2 + 3R_3 \)
    e) none of the above

16. The above matrix \( A \) (Question 14) in row echelon form is \( C \) (Question 15). Then \( A \)
    is the augmented matrix of a system of linear equations that has the following solutions:
    a) \( x = 1 \); \( y = 1 \); \( z = 1 \)
    b) \( x = 2 \); \( y = 0 \); \( z = 1 \)
    c) \( x = 5 \); \( y = -2 \); \( z = 1 \)
    d) This system has no solutions
    e) none of the above
17. If \[
\begin{bmatrix}
1 & 0 & -1 \\
0 & 1 & 2 \\
0 & 0 & 0
\end{bmatrix}
\] is the RREF of a system of equations, what is (are) the solution(s)?
   a) This system has no solutions
   b) \( x = -2 + 3t \), \( y = 2 - t \), \( z = t \); any real \( t \).
   c) \( x = 2 \), \( y = -1 \), \( z = 0 \) is a unique solution
   d) \( x = 2 + t \), \( y = -1 - 2t \), \( z = t \); any real \( t \).
   e) \( x = t \), \( y = 2 + t \), \( z = t \); any real \( t \).

18. If \[
\begin{bmatrix}
1 & 2 & 0 & 3
\end{bmatrix}
\] is RREF of a system of equations, what is (are) the solution(s)?
   a) \( x = 3 \), \( y = 2 \), \( z = 1 \).
   b) This system has no solutions.
   c) \( x = 3 - 2t \), \( y = t \), \( z = 2 \); any real \( t \).
   d) \( x = 3 \), \( z = 2 \) and \( y \) is undefined.
   e) none of the above.

Find the inverse of \[
\begin{bmatrix}
1 & 2 & 1 \\
0 & 0 & 1 \\
2 & 5 & 3
\end{bmatrix}
\] and use it to answer the next four questions.

19. What is the \((1, 2)\) position of \( \mathbf{M}^{-1} \)?
   a) 1
   b) 0
   c) 7
   d) \(-2\)
   e) \(-4\)

20. What is the \((2, 3)\) position of \( \mathbf{M}^{-1} \)?
   a) \(-4\)
   b) 7
   c) 0
   d) 2
   e) 1

21. What is the \((3, 2)\) position of \( \mathbf{M}^{-1} \)?
   a) \(-4\)
   b) 7
   c) 0
   d) 2
   e) 1

22. If \[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\] and \[
\begin{bmatrix}
1 \\
0
\end{bmatrix}
\] are the solutions to the linear system \( \mathbf{M} \mathbf{x} = \mathbf{B} \), what are the
   a) \( x = 7 \), \( y = -4 \), \( z = 1 \)
   b) \( x = 9 \), \( y = -5 \), \( z = 2 \)
   c) \( x = 0 \), \( y = 2 \), \( z = 14 \)
   d) the system has no solutions.
   e) none of the above.

23. What is \( 11 \) converted to base \( 2 \)?
   a) \((1101)_{2}\)
   b) \((10110)_{2}\)
   c) \((1011)_{2}\)
   d) \((211)_{2}\)
   e) \((10001)_{2}\)

24. What is \((24)_{7}\) in decimal?
   a) 18
   b) 86
   c) 33
   d) 10
   e) none of the above

25. What is the greatest common divisor of 91 and 104?
   a) 7
   b) 3
   c) 13
   d) 1
   e) none of the above

26. What is the smallest positive number in the residue class of 23 mod 7?
   a) 2
   b) \(-5\)
   c) 4
   d) 3
   e) 9

27. Does \( 5 \) have an inverse mod 17?
   a) yes
   b) no

28. Does \( 12x \equiv 1 \mod 21 \) have a solution?
   a) yes
   b) no
Part B (Long Answer, Answer all questions)

1. a) Given $G_1$

   What are the degrees of each node?

<table>
<thead>
<tr>
<th>node</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>degree</td>
<td></td>
<td></td>
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</tbody>
</table>

   Does $G_1$ have an Euler circuit? Why or why not?
   Find one if it exists.

[Graph of $G_1$]

Does $G_1$ have an Euler path? Why or why not?
Find one if it exists.

Find a Hamilton circuit of $G_1$ that starts at B.
(b) Given G2, where nodes and edges are both labeled with numbers:

What is the incidence matrix of G2?

What is the adjacency matrix of G2?

Use the adjacency matrix of G2 to calculate the number of two edge paths from node 3 to node 5.

(Show your work clearly, but only do as much calculation as is needed to answer the question).

List all the paths from node 3 to node 5 that consist of two edges.
2. (a) The Terrific Table Company produces two kinds of tables, fancy and plain. The fancy table takes 6 hours to cut material, and 2 hours to assemble. Plain tables take 2 hours to cut and 3 ours to assemble. Each table takes 1 hour to stain.

The Terrific Table Company makes $45 profit on every fancy table and $30 profit on every plain table. Each day there are 200 hours of cutting available, 70 hours of assembling available and 35 ours available for staining.

Give the formulation of this as a linear programming problem, BUT DO NOT SOLVE. (Be sure to define variables and give the objective function as well as the constraints)
(b) Find the maximum value of $P = 25x + 20y$ subject to the following constraints:

\[
\begin{align*}
    x + y & \leq 6 \\
    3x + 2y & \leq 13 \\
    2x + y & \leq 8 \\
    x & \geq 0 \\
    y & \geq 0
\end{align*}
\]