INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. Please show your work clearly.

No texts, notes, or other aids are permitted. Cellphones, electronic translators or any other type of electronic device are not permitted. However you may use simple calculators with one line display.

This exam has five parts. In Part A, answer all the multiple choice questions and record your answers in the box on the top of page 2. All other parts are long answer, and each long answer question should be answered in the space provided. There is 1 blank page for rough work. You may remove the blank page if you want, but be careful not to loosen the staple.

The value of each multiple choice question is 2 points. The value of each of the other questions is indicated in the left hand margin beside the statement of the question. The total value of all questions is 100 points.

Answer all questions on the exam paper.

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Part A: Multiple Choice Questions

Questions 1-10 carry 2 marks each. Write the letter corresponding to your choice of the answer in the box below. No partial marks will be given for Questions 1-10.

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1. What is the solution of the equation $4(2 - x) + 3(2x - 1) + 7 = 0$?
   (a) 1  (b) 6  (c) -6  (d) -1  (e) None of these

2. What is the slope of the line $2y + \frac{x-1}{4} = 0$?
   (a) $\frac{1}{4}$  (b) $-\frac{1}{4}$  (c) $\frac{1}{2}$  (d) $-\frac{1}{8}$  (e) $-\frac{1}{2}$

3. What is the slope-intercept form of the equation of the line through the point $\left(\frac{1}{3}, 0\right)$ and parallel to the line $x = \frac{3y}{3} - 9$?
   (a) $y = x - \frac{1}{3}$  (b) $y = 3x - 1$  (c) $y = \frac{1}{3}x - 3$  (d) $y = \frac{1}{3}x - 1$
   (e) $y = 3x + 3$

4. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 5 & 1 \end{bmatrix}$. What is the product $AB$?
   (a) $\begin{bmatrix} 9 & 2 \\ 1 & 0 \end{bmatrix}$  (b) $\begin{bmatrix} 9 & 1 \\ 1 & -8 \end{bmatrix}$  (c) $\begin{bmatrix} 9 & 2 \\ 1 & 0 \end{bmatrix}$  (d) $\begin{bmatrix} -9 & -1 & 8 \\ 2 & 0 & -1 \end{bmatrix}$
   (e) $AB$ is not defined.

5. Let $A = \begin{bmatrix} -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 \\ 1 & 4 \end{bmatrix}$. What is the product $BA^T$?
   (a) $\begin{bmatrix} -4 & 1 \end{bmatrix}$  (b) $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$  (c) $\begin{bmatrix} 1 & -4 \end{bmatrix}$  (d) $\begin{bmatrix} -5 \\ -11 \end{bmatrix}$
   (e) $BA^T$ is not defined.

6. Let $A = \begin{bmatrix} 3 & -5 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 \\ 3 & -3 \end{bmatrix}$. What is the sum $2A^T + 3B$?
   (a) $\begin{bmatrix} 3 & -6 \\ 19 & -7 \end{bmatrix}$  (b) $\begin{bmatrix} 3 & -16 \\ 9 & -7 \end{bmatrix}$  (c) $\begin{bmatrix} -3 & 6 \\ -1 & -7 \end{bmatrix}$  (d) $\begin{bmatrix} 3 & -6 \\ -1 & -7 \end{bmatrix}$
   (e) $2A^T + 3B$ is not defined.

7. If $A = \begin{bmatrix} 4 & -2 \\ -1 & 1 \end{bmatrix}$, then $A^{-1}$ is:
   (a) $\begin{bmatrix} -\frac{1}{2} & 1 \\ -\frac{1}{2} & 2 \end{bmatrix}$  (b) $\begin{bmatrix} \frac{1}{2} & 1 \\ \frac{1}{2} & 2 \end{bmatrix}$  (c) $\begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$  (d) $\begin{bmatrix} \frac{1}{2} & 1 \\ -\frac{1}{2} & 2 \end{bmatrix}$
   (e) $2A^T + 3B$ is not defined.
8. If the augmented matrix of a linear system is \[
\begin{bmatrix}
1 & 3 & \mid & 2 \\
0 & a - 1 & \mid & b + 2
\end{bmatrix},
\]
then the system has infinitely many solutions if:

(a) \(a = 1,\ b = -2\)  
(b) \(a = 0,\ b = 0\)  
(c) \(a = 1,\ b = 2\)  
(d) \(a = -1,\ b = 2\)  
(e) \(a\ and\ b\ are\ any\ real\ number.\)

9. A graph has degree set \(\{1, 2, 3, 3, 4, 5\}\). How many edges does it have?

(a) 9  
(b) 18  
(c) 36  
(d) 10  
(e) No such graph exists

10. How many months does it take for an investment to double if the interest rate is 6% compounded monthly?

(a) 141  
(b) 139  
(c) 137  
(d) Not enough information
Part B: Linear Systems

[12] 1. Consider the following linear system.

\[
\begin{align*}
  x - 2y + z + u &= 1 \\
  -x + 2y + u &= -1 \\
  2x - 4y + z &= 2 \\
\end{align*}
\]

(a) Find the augmented matrix of this system.

(b) Find the reduced row echelon form of the augmented matrix (RREF).

(c) Find all solutions of this system. Show your work to the left below, and write the solutions on the lines to the right.

\[
\begin{align*}
  x &= \quad \\
  y &= \quad \\
  z &= \quad \\
  u &= \\
\end{align*}
\]
Part C: Linear Programming

[8] 1. Formulate the following as a linear programming problem. Label your variables clearly, write down all the constraints and the objective function. **Set up but do not solve.**

A nutritionist is planning a menu that includes portions of two types of foods: foods $A$ and $B$. Each ounce of food $A$ contains 2 units of protein, 1 unit of iron, and 1 unit of thiamine; each ounce of food $B$ contains 1 unit of protein, 1 unit of iron, and 3 units of thiamine. Each ounce of $A$ costs 30 cents, while each ounce of $B$ costs 40 cents. The nutritionist wants the meal to provide at least 12 units of protein, at least 9 units of iron, and at least 15 units of thiamine. How many ounces of each of the foods should be used to minimize the cost of the meal?
2. Maximize the objective function $Z = 7x + 5y$, subject to the following constraints.

\[
\begin{align*}
x + y & \geq 2 \\
6x + 5y & \leq 30 \\
x & \leq 3 \\
x & \geq 0 \\
y & \geq 0
\end{align*}
\]

Sketch the feasible region, and find all the corner points.
Part D: Graph Theory

Consider the following graph:

[2] 1. What is the degree set for this graph (write in increasing order).

[2] 2. Does this graph contain an Euler circuit starting at node $E$? If so, produce one. If not, explain why not.

[4] 3. Does this graph contain an Euler path starting at node $E$? If so, produce one. If not, explain why not.

[4] 4. Does this graph have a Hamilton circuit starting from node $E$? If so, produce one. If not, explain why not.
5. Find 4 distinct paths from node $A$ to node $B$ using any number of edges.

6. Find the adjacency matrix for this graph.

7. Using the adjacency matrix, how many distinct routes with exactly 2 edges are there from node $A$ to node $B$? Show your work. Find all such routes.

8. Using the adjacency matrix, how many distinct routes with exactly 2 edges are there from node $A$ back to itself? Show your work. Find all such routes.
Part E: Finance

\[

t_n = P \left(1 + \frac{1}{100m}\right), \quad C_n = P \left(1 + \frac{i}{100m}\right)^n, \quad A_n = \frac{P \left[1 + \left(1 + \frac{i}{100m}\right)^n - 1\right]}{100m}
\]

[7] 1. $2000$ is invested as simple interest calculated quarterly with interest rate $8\%$ for 2 years. How much more interest can be earned if it is compound interest compounded quarterly with interest rate $8\%$ for 2 years?
2. A student wishes to set up an annuity that will have $10,000 after 4 years. Determine the monthly payment if the interest rate is 5% compounded monthly.

3. A couple take out a loan of certain amount at 6% amortized over 20 years calculated monthly. In order to pay back the loan, they pay $600 per month. Determine the remaining principle after 12 years. (Hint: You do not need to calculate the amount of loan.)