MATH 1010 Midterm B March 12, 2008

Instructions:

1. Answer all questions on the machine-scored answer sheet provided. Use pencil only.
2. Return examination paper with machine-scored answer sheet.
3. Single-line display calculators permitted. No other aids permitted.
4. Fill in the information requested below.
5. The examination invigilators may not interpret or explain questions to you.
6. Fill in your student number on the machine-scored sheet and ***encode*** it as well.

FAMILY NAME _____ **FIRST NAME** _____

STUDENT NUMBER _____

SIGNATURE _____

INSTRUCTOR _____

Use the following matrices to answer questions 1-5 only :

Let $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ -1 & 1 \\ 0 & 0 \end{bmatrix}$, $C^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$, $D = \begin{bmatrix} -4 \\ -1 \end{bmatrix}$ **and** $E = \begin{bmatrix} -1 & 0 & 2 \end{bmatrix}$.

1. What is the product BA?

   (a) $\begin{bmatrix} 2 & -1 & 0 \\ 1 & 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 1 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 1 \\ -1 & 2 \\ 0 & 0 \end{bmatrix}$

   (e) It is not defined.

2. What is A^2?

   a) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ (e) It is not defined.

3. What is $2A + DE$?

   (a) $\begin{bmatrix} 6 & -2 & -8 \\ 1 & 2 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -2 \\ 1 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 \\ 6 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & -2 \\ 6 & -2 & -8 \end{bmatrix}$

   (e) It is not defined.

4. What is $D^T A + EB$?

   (a) $\begin{bmatrix} 0 & -6 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} -6 \\ -6 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ -6 \end{bmatrix}$

   (e) It is not defined.

5. How many of the expressions $AC^{-1} + B$, $DE + B^T$, BB^T, and A^3 are defined?

   (a) 1 (b) 2 (c) 3 (d) All of them (e) None of them

6. How many of the following matrices are in **Row Echelon Form**?

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(a) 1 (b) 2 (c) 3 (d) 4 (e) None of these

7. If the reduced row echelon form of the augmented matrix of a linear system is $\left[\begin{array}{ccc|c} 1 & 0 & 3 & -4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$, which one of the following is **incorrect**?

(a) The system has 4 equations. (b) The system has 4 equations and 3 variables.
(c) The system has infinitely many solutions. (d) There are 3 leading one.

8. If the reduced row echelon form of the augmented matrix of a linear system is $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$, then the solutions to the linear system are:

(a) $x = 3 + 2z,\ y = 0$ and z arbitrary (b) $x = 3 - 2z,\ y = 0$ and z arbitrary
(c) $x = 2z - 3,\ y = 0$ and z arbitrary (d) there is no solution

9. If $\left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$ is the reduced row echelon form of the augmented matrix of a system, how many solutions does the system have ?

(a) no solution (b) only one solution (c) infinitely many solutions
(d) exactly two solutions (e) None of these

10. If the reduced row echelon form of the augmented matrix of a linear system contains a row of zeros, then the system has:

(a) only one solution (b) no solution at all (c) exactly two solutions
(d) either no solution, one solution or infinitely many solutions (e) none of these

11. If the augmented matrix of a linear system is $\left[\begin{array}{cc|c} 1 & -6 & 7 \\ 0 & k-3 & 1 \end{array}\right]$, then the system has no solution if:

(a) $k = 4$ (b) $k = 0$ (c) $k = 3$ (d) $k = 2$ (e) k is any real number

12. What elementary row operations are used on the matrix $\left[\begin{array}{ccc|c} 2 & -1 & 3 & 0 \\ 1 & 0 & 1 & -1 \\ 1 & 1 & -1 & 2 \end{array}\right]$ to obtain the matrix $\left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & -1 & 1 & 2 \\ 1 & 1 & -1 & 2 \end{array}\right]$?

(a) $R_1 \leftrightarrow R_2$, then $R_2 \to -2R_1 + R_2$ (b) $R_1 \leftrightarrow R_3$, then $R_2 \to -2R_1 + R_2$
(c) $R_1 \leftrightarrow R_2$, then $R_2 \to 2R_1 + R_2$ (d) $R_1 \leftrightarrow R_2$, then $R_2 \to 2R_1 - R_2$

13. Let $A = \begin{bmatrix} -1 & 2 & 1 \\ 3 & -5 & -1 \\ -4 & 8 & 5 \end{bmatrix}$, if **only** elementary row operations $R_2 \to 3R_1 + R_2$ and $R_3 \to -4R_1 + R_3$ are performed on A, then the result is the matrix:

(a) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 2 & 1 \\ 0 & -11 & -4 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 16 & 9 \end{bmatrix}$

14. Which one of the following statements is **correct**?

(a) In row echelon form, you never get a row of zeros.
(b) In row echelon form, the second row starts with 2.
(c) In reduced row echelon form, always all rows start with 0.
(d) The reduced row echelon form of a matrix is unique.

15. Let $A = \begin{bmatrix} 4 & -1 \\ 6 & -2 \end{bmatrix}$, then A^{-1} is:

(a) $\begin{bmatrix} 1 & -\frac{1}{2} \\ 3 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & \frac{1}{2} \\ -3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{4} & -1 \\ \frac{1}{6} & -\frac{1}{2} \end{bmatrix}$ (d) $\begin{bmatrix} -2 & 1 \\ 6 & 4 \end{bmatrix}$ (e) none of these

16. Let $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & -1 & 1 \\ 2 & 5 & -1 \end{bmatrix}$, then (2, 1)-entry of A^{-1} is:

(a) -1 (b) 1 (c) 0 (d) $\frac{1}{2}$ (e) A has no inverse.

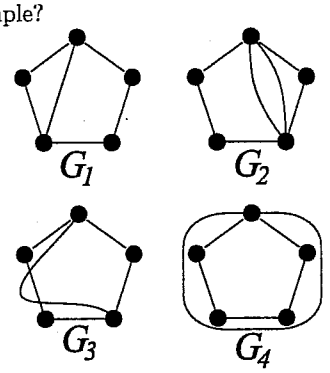
17. For the linear system $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, if $A^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 0 & -1 \\ 2 & -1 & 4 \end{bmatrix}$, then the solution of the system is:

(a) $x = -1,\ y = -1,\ z = 0$ (b) $x = 1,\ y = 1,\ z = 0$ (c) $x = 3,\ y = -1,\ z = 0$
(d) $x = -1,\ y = 1,\ z = 4$ (e) There is not enough information to solve the system.

18. If $A^T = \begin{bmatrix} -1 & 4 \\ 0 & -1 \end{bmatrix}$ and $B^T = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$, then $(AB)^T$ is:

(a) $\begin{bmatrix} -2 & 8 \\ -1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 4 \\ -1 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & -8 \\ 1 & -3 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -1 \\ 2 & 4 \end{bmatrix}$

19. Which of the following graphs are simple?



(a) G_1 only (b) G_1 and G_3 only (c) G_1, G_3, and G_4, but not G_2.
(d) All of them are simple. (e) None of them are simple.

20. How many vertices does a simple graph have if it has 18 edges, and each vertex has degree 3?

(a) 3 (b) 6 (c) 12 (d) 15 (e) None of these

21. How many vertices does a simple graph have if it has 18 edges, 3 vertices of degree 4, and all other vertices have degree 6?

(a) 3 (b) 4 (c) 6 (d) 7 (e) None of these

22. Is there a graph with the degree set $\{1, 1, 2, 2, 4\}$?

(a) Yes, in fact there is a simple graph with this degree set.
(b) Yes, but such a graph must be non-simple.
(c) No
(d) Not enough information to tell.

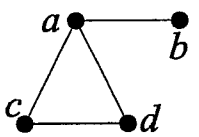
23. Is there a simple graph with the degree set $\{2, 2, 2, 3, 5\}$?

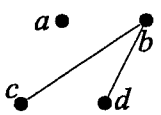
(a) Yes, and such a simple graph has 7 edges.
(b) Yes, and such a simple graph has 14 edges.
(c) No
(d) Not enough information to tell.

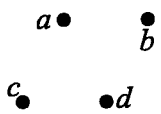
24. How many simple graphs are there with exactly two vertices?

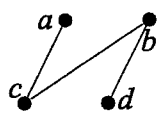
(a) 0 (b) 1 (c) 2 (d) 3 (e) more than 3

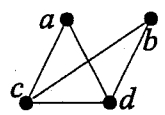
25. For any simple graph G, the ***complement*** of G is the graph with the same vertices as G, but contains only those edges that are missing in G. What is the ***complement*** of the following graph:



(a) 

(b) 

(c) 

(d) 

(e) None of the above