DEPARTMENT OF MATHEMATICS
UNIVERSITY OF MANITOBA

FA/MATH 1020 Math In Art
Final Exam, 15 Dec 2007
Paper # 496

Instructors
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Dr. Michelle Davidson

LAST NAME: (Print in ink) ______________________________

FIRST NAME: (Print in ink) ______________________________

STUDENT NUMBER: ______________________________

SIGNATURE: ____________________________________________
(I understand that cheating is a serious offence)

You may use calculators and drawing instruments like the ruler and the compass. However, show all the intermediate steps and calculations.

Important: “Construct” means “construct using an unmarked ruler and a compass”. The phrase “unmarked ruler” stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do not erase them). Use words to describe BRIEFLY what you have done.

Cell phones or other aids are not allowed.

The exam has a total of 10 pages (including this title page) and two blank sheets for rough work. Please check that you have all the pages.

Please do not write in this column

1. ___________/8
2. ___________/4
3. ___________/8
4. ___________/8
5. ___________/10
6. ___________/10
7. ___________/10
8. ___________/12

Total ____________/70
1. In each of the following, numbers in the first column indicate the lengths of the three sides of a triangle. Determine the list of properties that are valid for the triangle in question.

<table>
<thead>
<tr>
<th>triangle with sides a, b, c</th>
<th>equilateral triangle</th>
<th>isosceles triangle</th>
<th>golden triangle</th>
<th>right-angled triangle</th>
<th>no such triangle exists</th>
</tr>
</thead>
<tbody>
<tr>
<td>6, 6, 6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5, 12, 13</td>
<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>3, 3, 6</td>
<td></td>
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<tr>
<td>$1 + \sqrt{5}$, 2, 2</td>
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<tr>
<td>5, 5, $\sqrt{50}$</td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

2. You are given below two line segments AB and CD. The length of AB is one unit and the length of CD is 5 units. Using ruler and compass, draw a line segment of length $\sqrt{3}$.

A __ ____________________________ C

D
3 (a). List the symmetries of each of the following designs. Use the notation refl $\lambda$ (for mirror reflection about the line $\lambda$), rot(C, $\theta$) (for rotational symmetry with centre C in degrees $\theta$ counterclockwise).

<table>
<thead>
<tr>
<th>Design</th>
<th>symmetries</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image.png" alt="Design Image" /></td>
<td><img src="image.png" alt="Symmetries Image" /></td>
</tr>
</tbody>
</table>

3 (b) You are given that a finite planar design D has *exactly five* symmetries including the id. Determine all the five symmetries of D. Draw a picture of such a design D.
4. Using the technique of drawing straight lines as tangents, construct an *asymmetrical* parabola-shaped ceiling. The "curved" roof will be straight lines creating an illusion of a parabola. You are given the base line and the two roof lines (A and B below).

roof line A: ________ choose any length from 3' to 3.5"

roof line B: ________ choose any length from 5.5" to 6"

given base line
Complete this table for the five Platonic solids.

<table>
<thead>
<tr>
<th>Name of the solid</th>
<th>V = # of Vertices</th>
<th>E = # of Edges</th>
<th>F = # of Faces</th>
<th>shape of each Face</th>
<th># of Faces at each Vertex</th>
<th>V-E+F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tetrahedron</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Cube</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Octahedron</td>
<td>6</td>
<td>12</td>
<td>8</td>
<td>triangle</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Icosahedron</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>Dodecahedron</td>
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</tr>
</tbody>
</table>

State the Euler’s theorem connecting the vertices, edges and faces of a convex polyhedra.

Determine the values of V, E and F for the polyhedron shown here.
Find the value of V – E + F.
Does this contradict the Euler’s Theorem? Give reasons for your answer.
The butterfly in the image B is obtained from image A by rot(C, \( \theta \)), a rotation of the plane around C by an angle of \( \theta^\circ \). Locate the point C, the centre of rotation and find \( \theta \), the angle of rotation.
7. Reproduce a perspective image of the fish drawn on a square-tiled pavement (given on the right side) onto the rectangular wall given below. Also locate the vanishing point of the perspective drawing (of the rectangular wall).
On the right hand side you are given a line drawing of a stylistic artwork of a butterfly drawn in the usual Euclidean plane. Reproduce this art on the hyperbolic canvas given below.

In your new drawing, the Euclidean lines will be redrawn as hyperbolic straight lines.

(Here the point O is the centre of the hyperbolic canvas)