INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam.

You may use an unmarked ruler and compass. No texts, notes, cell phones, or other aids are permitted.

This exam has a title page, 7 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but do not remove the staples.

The value of each question is indicated in the table to the right. The total value of all questions is 70.

Answer all questions on the exam paper in the space provided beneath the question.

In the construction problems, do not erase the intermediate lines, or arcs of circles.

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Important: The term "construct" in all of the questions means “construct using an unmarked ruler and a compass”. The phrase “unmarked ruler” stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do not erase them). Use words to describe BRIEFLY what you have done.

[8] 1. (a) Construct the center of the circle given below.

(b) Construct one circle that passes through the point P that is perpendicular to the circle given below.

[9] 2. Construct a golden rectangle to that the line segment shown below is the height of that rectangle.
3. (a) **Construct** the image of the triangle given below under the rotation centered at the given point $O$ and through an angle of $45^\circ$. (Note: you would first need to **construct** an angle of $45^\circ$.)

(b) The point $A'$ shown in the picture below is the image of the point $A$ under a reflection $rgf(m)$ with respect to a line $m$ (not shown). Construct the line $m$ and then find the image of the point $B$ under the reflection $rgf(m)$. 

\[ A' \quad B \]
4. 

(a) Find the group of the symmetries of the following object. Clearly identify the symmetries.

(b) Find the group of the symmetries of the following Frieze pattern. Clearly identify the symmetries.

(c) Draw an example of a design having exactly four symmetries (including id).
5. The picture below depicts a square drawn in two-point perspective.

(a) Find the horizon and the vanishing points.

(b) Construct a 2 x 2 chessboard in the given square in the given perspective drawing. (That is, subdivide in the perspective drawing the given square into 4 equal smaller squares.)
6. In the two figures below (Figure 1 and Figure 2) we show the first two steps in the construction of a fractal.

(a) Draw the figure representing the next step in the construction of the fractal. (The dot in the middle of the large circle in Figure 1 represents the center of that circle. You do NOT need to precisely construct the circles and the lines in the next step.)

(b) The final fractal $F$ will be constructed after infinitely many steps (the first few of them are described in Figures 1, 2 and in the correct solution to question (a) here). Find a central similarity of stretching factor not equal to 1 that will send the fractal $F$ into itself. (To get full marks here, you need to indicate in the figure you draw in part (a) where the center of the central similarity is, and you need to state a specific number for the stretching factor of that central similarity.)
7. We are given a hyperbolic line \( l \) and a point \( A \) on that hyperbolic line.

(a) Construct one hyperbolic line parallel to \( l \).

(b) Construct the hyperbolic line passing through \( A \) and perpendicular to \( l \).
8. (a) Which of the letters D, S, B and P are homotopic?

(b) Show that the letters A and R are homotopic by drawing at least three in-between sketches showing how A can be continuously deformed into R.