April 20, 2005

FINAL EXAMINATION
TITLE PAGE
TIME: 2 HOURS

THE UNIVERSITY OF MANITOBA

LAST NAME: (Print in ink)

FIRST NAME: (Print in ink)

STUDENT NUMBER:

SEAT NUMBER:

SIGNATURE (in ink): (I understand that cheating is a serious offense)

Please mark your section:

☐  L02  Tues, Thurs  8:30 - 10:00  S. Kalajdzievski

☐  L03  Tues, Thurs  8:30 - 10:00  R. Padmanabhan

DO NOT WRITE IN THIS COLUMN

1. ________/10
2. ________/10
3. ________/10
4. ________/10
5. ________/10
6. ________/10
7. ________/10
8. ________/10
9. ________/10
TOTAL ________/90

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. Please show your work clearly.

Electronic calculators with single line display permitted. Cell phones or other aids are not permitted.

This exam has a title page, 9 pages of questions and 2 blank pages at the end for rough work. Please check that you have all the pages.

The value of each question is indicated at the bottom of each question. The total value of all questions is 90.

Answer all questions on the exam paper in the space provide beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Important: "Construct" means "construct using an unmarked ruler and a compass". The phrase "unmarked ruler" stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do not erase them). Use words to describe BRIEFLY what you have done. The golden ratio \( \phi \) is approximately 1.618 and you may assume that \( \phi = 1.618 \) in the computations below.
1. (a) Construct the unique straight line through the given point P and perpendicular to a AB.
(b) Construct a straight line m through P and parallel to AB.

   P

   A _________________ B
   5+5=10

2. The triangle ABC is an acute golden triangle with a base AB equal to a centimeters, the side AC equal to b centimeters, and the height h of the triangle equal to 30776 cm.
(a) What is the ratio \( \frac{h}{a} \)?
(b) Find the length a of the base of the triangle.
3. (a) Determine the symmetries of the design given below.

(b) Determine all of the symmetries of the equilateral triangle given below.

(c) Draw an example of a design having exactly three symmetries (including id). Identify all the symmetries in your design.
4. Define a sequence of "gibbonacci" numbers $g_n$ as follows:

$$g_1 = 3, g_2 = 7$$

$$g_n = g_{n-1} + g_{n-2}, \ n = 3, 4, \ldots$$ (that is, starting from $g_3$ each of these numbers is the sum of the previous two).

(a) Write down the first 6 terms of the sequence of gibbonacci numbers.

(b) It can be shown that $g_{14} = 2063$ and that $g_{16} = 5401$. Compute $g_{15}$. Justify your answer (that is, show how you have found $g_{15}$).

(c) Use a calculator to find the values of $\frac{g_{15}}{g_{14}}$ and $\frac{g_{16}}{g_{15}}$. Then make a conjecture: As $n$ gets larger and larger, the ratio $\frac{g_{n+1}}{g_n}$ approaches ________

\[
2 + 6 + 2 = 10
\]
5. Construct a golden rectangle with AB as the longer side of the rectangle. Construct a smooth golden spiral within the rectangle.
6. Using the technique of constructing a parabola as an envelope of tangents, draw a parabolic antenna with the base lines as shown. (you may add the base support by freehand)

This part of the antenna needs to be constructed using tangents.
7. You are given (see the picture) the horizon line, two vanishing points VP1 and VP2 and two opposite corners A and B of a square (in perspective drawing).

(a) Construct a two point perspective drawing of the (rest of the) square (with A and B being its opposite corners).

(b) Subdivide (tile) the square constructed in (a) into 4 equal smaller squares (in the perspective drawing done in part (a)).
8. We are given a hyperbolic line $l$ and a point $A$ in the Poincare model of the hyperbolic geometry (please see the picture below).

Construct two hyperbolic lines through the point $A$ and parallel to the given hyperbolic line $l$. Leave the intermediate constructions (do not erase them); briefly explain your steps.
9. (a) Compute the Euler characteristic of the octahedron (see the picture).

(b) What is the genus of the surface of the key depicted below? (Note that the key is not flat — it is a 3-dimensional object).
(c) Show that the two objects given below are homotopically (topologically) equivalent by drawing at least 3 in-between sketches showing how one can be continuously deformed into the other.

4 + 2 + 4 = 10