136.102 Math in Art  
Midterm Exam  
February 21, 2006

Name:__________________________  Student Number ____________

*Important:* "Construct" means "construct using an unmarked ruler and a compass". The phrase "unmarked ruler" stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do not erase them). Use words to describe BRIEFLY what you have done.

1. (a) Construct the line passing through the point $A$ and parallel to the given line $l$.

![Diagram](image1.png)

(b) Construct a line passing through the given point $A$ (in the illustration shown below) and intersecting the given line $l$ at the angle of 45°.

![Diagram](image2.png)
2. (a) Construct a regular pentagon over the given line segment (as one of the sides of the pentagon).

(b) Construct an angle of 144°.
3. (a) What are Fibonacci numbers? (Write down the definition.)

(b) It can be verified that $f_{22} = f_{24} + 28657$ and that $f_{22} = 17711$ (no need to check that). Find $f_{24}$.
(In this question, as usual, $f_n$ is the $n$-th Fibonacci number in the list of Fibonacci numbers.)

4. Find the group of symmetries for each of the three objects shown below. If you claim a rotational symmetry, indicate the center of the rotation and the angle of rotation. If there are reflections, show the line of reflection. If there are translational symmetries show or describe the vectors of translation.

<table>
<thead>
<tr>
<th>OBJECT</th>
<th>THE GROUP OF SYMMETRIES</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Object 1" /></td>
<td></td>
</tr>
<tr>
<td><img src="image2.png" alt="Object 2" /></td>
<td></td>
</tr>
</tbody>
</table>
5. Suppose the point $f(A)$ is the image of the point $A$ and the point $f(B)$ is the image of the point $B$ under the central symmetry $f$. Find (construct) the center of the central symmetry $f$ and then construct the image $f(C)$ of the point $C$ (as shown in the illustration) under the central symmetry $f$. 