

**DEPARTMENT OF MATHEMATICS
UNIVERSITY OF MANITOBA**

**FA/MATH 1020 Math In Art
Final Exam, 16 April 2007
Paper # 252**

Examiners

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LAST NAME: (Print in ink) _____

FIRST NAME: (Print in ink) _____

STUDENT NUMBER: _____

SIGNATURE: _____

(I understand that cheating is a serious offence)

Please circle the name of your Math Instructor

You may use calculators and drawing instruments like the ruler and the compass. However, show all the intermediate steps and calculations.

Important: "Construct" means "construct using an unmarked ruler and a compass". The phrase "unmarked ruler" stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do not erase them). Use words to describe BRIEFLY what you have done. The golden ratio ϕ is approximately 1.618 and you may assume that $\phi = 1.618$ in the computations below.

Cell phones or other aids are not allowed.

The exam has a total of 10 pages (including this title page) and two blank sheets for rough work. Please check that you have all the pages.

**Please do not write
in this column**

1. _____/5

2. _____/5

3. _____/6

4. _____/7

5. _____/10

6. _____/10

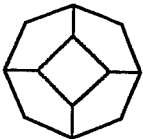
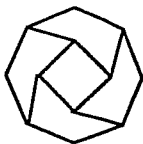
7. _____/12

8. _____/15

Total _____/70

1. (a) List all the symmetries of the two finite planar designs given below
For mirror reflections, mention the corresponding lines of reflection
For rotations, mention the corresponding centre and angles of rotation.

Design	symmetries
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5

2. Recall that a glide reflection is a symmetry that first translates the plane and then reflects it with respect to a line parallel to the direction of the translation vector. The following design was created by applying a glide reflection to the left-most "R" (stage 1), to get the middle "R" (stage 2) and then by applying the same glide reflection to the middle "R" (stage 2) to get the right-most copy of "R" (stage 3).

Find the reflection line. Draw the next stage of the basic design R in this strip pattern.

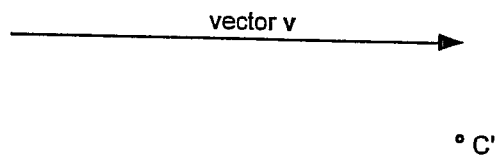
stage 1
R

stage 3
R

stage 2
K

5

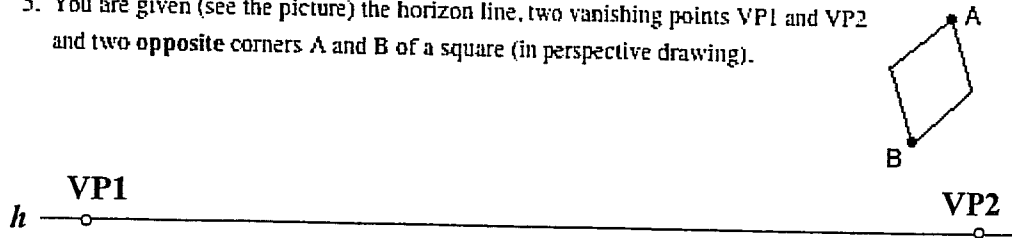
3. The points A' , B' and C' shown below are obtained by translating a set of three points A , B and C through the vector v . The original three points A , B and C were all points on a circle. Construct the centre of that circle.



4. Construct a golden rectangle with AB as the longer side of the rectangle.
Construct a smooth golden spiral within the rectangle.

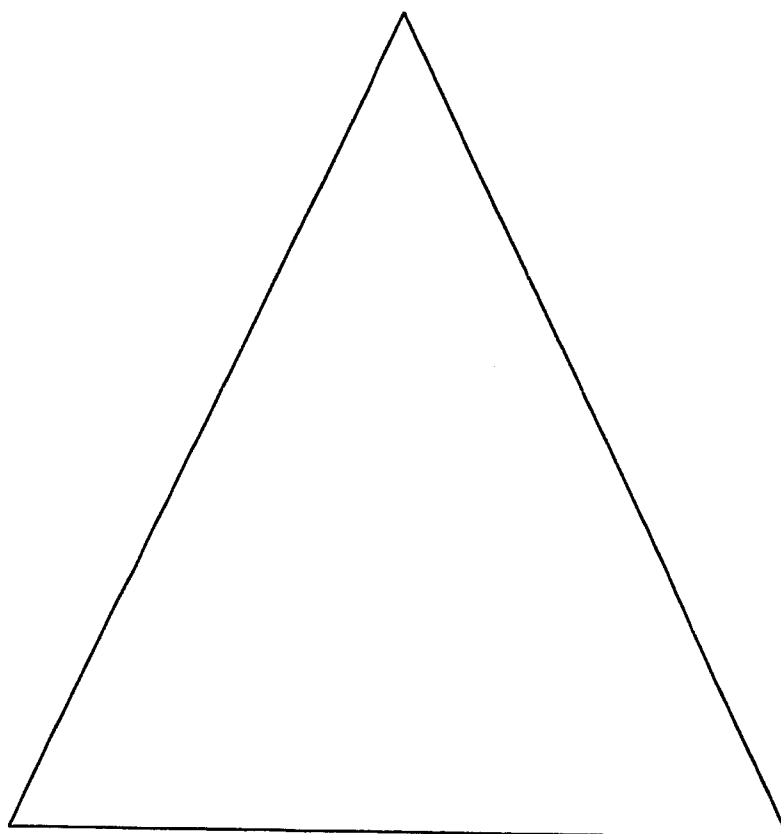
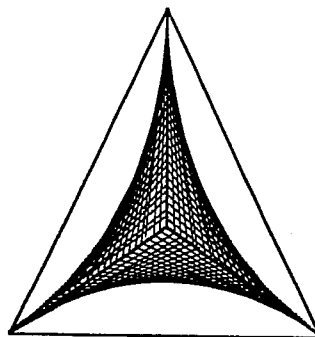


5. You are given (see the picture) the horizon line, two vanishing points VP1 and VP2 and two **opposite** corners A and B of a square (in perspective drawing).



- (a) Construct a two point perspective drawing of the (rest of the) square (with the points A and B being its opposite corners).
- (b) Subdivide (or tile) the square constructed in (a) into 16 **equal** smaller squares (in the perspective drawing as done in part (a)).

6. Using the technique of drawing parabolas with tangents, reproduce the string art given on the right in the triangular area given below.



7. (a) Give the Euler's formula connecting V , E and F for a convex polyhedron (recall V = # of vertices, E = # of edges and F = # of faces).
- (b) Let P be the convex polyhedron obtained by glueing together two identical regular tetrahedra along one face (to obtain a triangular dipyramid). Find the values of V , E and F for this polyhedron. Calculate the value of $V-E+F$.
- (c) Is this a regular polyhedron? Give reasons for your answer.

8. Hyperbolic Plane Constructions.

- (a) Draw the hyperbolic line joining the two points C and D.
- (b) Draw the hyperbolic line joining the two points A and B.
- (c) Draw two hyperbolic lines through the point D and parallel to AB.
- (d) How many such lines can be drawn through D and parallel to AB?

Note that the point C is the centre of the horizon circle.

