DATE: October 23, 2006

DEPARTMENT & COURSE NO: MATH 1300


EXAMINER: Various

NAME: (PRINT) _______________________________________

STUDENT NUMBER: __________________________

SIGNATURE: _______________________________________

(I understand that cheating is a serious offense)

Please mark your section number.

☐ Section A01 MWF (9:30 – 10:20) R.S.D. Thomas
☐ Section A02 T & R (8:30 – 9:45) C.K. Gupta
☐ Section A03 MWF (1:30 – 2:20) Y. Zhang
☐ Section A04 T & R (11:30 – 12:45) N. Zorboska
☐ Section A91 Challenge for Credit SJR

INSTRUCTIONS TO CANDIDATES:

This is a 1 hour exam. Please show your work clearly. Please justify your answers, unless otherwise stated.

No calculators or other aids are permitted.

This exam has a title page, 6 pages of questions and 1 blank page for rough work. Please check that you have all the pages.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 60.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

DO NOT WRITE IN THIS COLUMN

1. _____  / 8
2. _____  / 8
3. _____  / 12
4. _____  / 12
5. _____  / 17
6. _____  / 6
7. _____  / 17

TOTAL  / 60
Values

1. Solve, by Gauss-Jordan elimination, the linear system:

\[
\begin{align*}
  x - y - z &= -1 \\
  -x + y + 2z &= 2 \\
  2x - 2y + z &= 1
\end{align*}
\]
VALUES

[8] 2. Let \( A = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 3 & 4 \end{bmatrix} \) and \( C = \begin{bmatrix} 1 & -1 \\ 0 & -1 \\ 1 & -3 \end{bmatrix} \).

In each of the following cases, compute the given expression or briefly explain why the expression cannot be calculated:

a) \( AB \)

b) \( A + B \)

c) \( B + 2C^T \)

d) \( AB - BA \)
3. Let \( A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \)

a) Find \( A^{-1} \).

b) Use (a) to solve the system \( Ax = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \), where \( x \) is a column matrix of variables.
Values

\[ 3A^{-1} = \begin{bmatrix} 0 & 3 \\ 3 & 6 \end{bmatrix}. \]

a) Find \( A \).

b) If \( B \) is derived from \( A \) by adding \(-2\) times row two to row one \( (A \rightarrow R_1 \rightarrow 2R_2 + R_1 \rightarrow B) \), find the elementary matrices \( E \) and \( F \) such that \( B = EA \) and \( A = FB \).
Values

\[
A = \begin{bmatrix}
1 & 2 & 3 \\
2 & 0 & a \\
1 & 4 & 1
\end{bmatrix}
\]

(7) 5. Let \( A = \begin{bmatrix}
1 & 2 & 3 \\
2 & 0 & a \\
1 & 4 & 1
\end{bmatrix} \)

(a) Evaluate \( \det(A) \) by expansion along column 2. No other method will be awarded marks. Show all your work.

b) For what value of \( a \) is \( A \) invertible?

[6] 6. Evaluate \( \det \begin{bmatrix}
0 & 2 & -2 \\
1 & 2 & 4 \\
1 & 3 & -1
\end{bmatrix} \) by row reduction to the determinant of an upper triangular matrix. No other method will be awarded marks. Show all your work.
Values

[7] 7. Let $A$ be a $4 \times 4$ matrix, such that $\det(A) = 2$.

a) Write the reduced row echelon form of $A$.

b) Find all of the solutions of the linear system $Ax = 0$ (where $x$ is a column matrix of variables, and $0$ is a column matrix of zeroes).

c) Find $\det(-A^T)$.

d) Find $\det(B)$ if you know that $\det(BA^{-1}) = 1$. 