UNIVERSITY OF MANITOBA

DATE: October 26, 2009

MIDTERM I
TITLE PAGE

COURSE: MATH 1300
EXAMINATION: Vector Geometry and Linear Algebra

TIME: 60 minutes
EXAMINER: Various

FAMILY NAME: (Print in ink) _____

GIVEN NAME(S): (Print in ink) _____

STUDENT NUMBER: _____

SIGNATURE: (in ink) _____

(I understand that cheating is a serious offense)

- ❑ A01 Dr. Michael Doob
- ❑ A02 Dr. Julien Arino
- ❑ A03 Dr. Michelle Davidson
- ❑ A04 Dr. David Gunderson
- ❑ A05 Dr. Julien Arino
- ❑ A91 Challenge for Credit

INSTRUCTIONS TO STUDENTS:

This is a 60 minute exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 4 pages of questions, and one blank page (which may be removed). Please check that you have all the pages.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 60 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	11	
2	2	
3	4	
4	3	
5	2	
6	8	
7	8	
8	2	
9	6	
10	7	
11	7	
Total:	60	

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[2] 1. (a) Define what is meant by a "linear equation in variables $x_1, ..., x_n$".

[2] (b) Define what it means for an $n \times n$ matrix A to be *invertible*.

[2] (c) What is a system of linear equations of the form $A\mathbf{x} = \mathbf{0}$ called?

[2] (d) Define what is meant by "A is symmetric". (Be brief!)

[3] (e) Give an example of an inconsistent system of linear equations.

[2] 2. For a system of equations of the form $A\mathbf{x} = \mathbf{0}$, is it
❑always consistent,
❑sometimes consistent,
❑or inconsistent?
Check the appropriate box and explain.

[4] 3. Let A be an $n \times n$ matrix. State four *additional* properties equivalent to "A is invertible".

[3] 4. Prove that if a matrix A has an inverse, then this inverse is unique. Give reasons for each step.

[2] 5. What is the maximum number of 0's in an invertible 5×5 matrix? Explain.

[7] 6. (a) By Gauss-Jordan elimination, solve the (consistent) system

$$\begin{array}{rcrcrcr} x & + & 2y & - & z & = & 3, \\ 4x & + & 10y & + & 2z & = & 10, \\ x & + & 3y & + & 2z & = & 2. \end{array}$$

If row operations are not clearly and properly identified, your answers will not be graded. If there is more than one solution, state the solution using parameter(s).

[1] (b) Find a particular solution and verify that it is indeed a solution.

[8] 7. Let $A = \begin{bmatrix} 4 & 0 & -1 \\ 1 & 3 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -3 \\ 0 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 3 & -2 \\ 0 & 0 \\ -1 & 7 \end{bmatrix}$.

Calculate each of the following, and if the expression is not defined, say why.

[2] (a) The size of $A^T C^T$=_____

[2] (b) AC.

[2] (c) B^{-1} (by any method).

[2] (d) $A^T C$.

[2] 8. For what values of a and b is the matrix $M = \begin{bmatrix} 1 & a \\ a & b \end{bmatrix}$ an elementary matrix?

9. Calculate the following determinants by any method; only answers are graded:

[1] (a) $\begin{vmatrix} 1 & -5 \\ 7 & 6 \end{vmatrix}$=_____.

[2] (b) $\begin{vmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}$=_____.

[3] (c) $\begin{vmatrix} 1 & 0 & -5 & 2 \\ 0 & 6 & 1 & 0 \\ 3 & -6 & -2 & -3 \\ 1 & 0 & 0 & -1 \end{vmatrix}$=_____.

[7] 10. Suppose that A is a 5×5 matrix and $\det(A) = -2$. Find (only answers are graded):

[2] (a) $\det(A^{-1})$_____;

[1] (b) $\det(A^4)$_____;

[2] (c) $\det(3A)$_____;

[2] (d) $\det(\operatorname{adj}(A))$_____.

[7] 11. Let $H = \begin{bmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{bmatrix}$. Given that $\det(H) = 12$, compute

[3] (a) $\operatorname{cof}(H)$, the cofactor matrix for H,

[2] (b) $\operatorname{adj}(H)$, the adjoint of H, and

[2] (c) H^{-1} (using the adjoint).