THE UNIVERSITY OF MANITOBA

DATE: February 23, ~~2005~~ 2006
PAPER NO
DEPARTMENT & COURSE NO: 136.130
EXAMINATION: Vector Geom. and Lin. Algebra

MIDTERM EXAMINATION
TITLE PAGE
TIME: 1 hour
EXAMINER: Various

NAME (print in ink): _____

Student Number: _____

Signature: _____
(I understand that cheating is a serious offense)

Identify your section:

	Section	Instructor	Slot	Time
☐	L05	K. Kopotun	5	TTh 10:00-11:15am
☐	L06	G. I. Moghaddam	8	MWF 1:30-2:20pm
☐	L07	G. I. Moghaddam	12	MWF 3:30-4:20pm
☐	L08	C. Platt	15	TTh 4:00-5:15pm
☐	L09	J. Sichler	E2	T 7:00-10:00pm

INSTRUCTIONS:

- This is a one hour exam consisting of 7 questions on 7 pages. Make sure you have a complete copy.
- Attempt as many problems as you can (bonus problem is optional).
- Note that you can achieve a score greater than 60.
- No texts, notes, calculators or other aids are permitted.
- Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.
- Show all your work and **justify** your answers. **Unjustified answers will receive LITTLE or NO CREDIT.**

Problem	Mark
1	/10
2	/10
3	/9
4	/10
5	/6
6	/9
7	/6
Total	/ 60
Bonus	/4

[10] 1. Consider the following linear system of equations.

$$\begin{cases} x + y - z = 1 \\ -2y + 4z = 6 \\ 3x + y + z = 9 \\ 3x - y + 5z = 15 \end{cases}$$

[7] (i) Find the reduced row-echelon form (RREF) of the augmented matrix. Make sure to indicate what row operations you are using.

[3] (ii) Using part (a), find all solutions of this system (*i.e.*, determine the solution set).

[10] 2. Consider the system

$$\begin{cases} x + y + 2z &= a \\ 2x + by + 4z &= 1 \end{cases}$$

In each case, determine all a and b which give the indicated number of solutions, if possible. If no such a and b exist, give the reason why not.

(i) no solutions

(ii) exactly one solution

(iii) exactly two solutions

(iv) infinitely many solutions

[9] 3. Which of the following statements are true. Start with "true" or "false" and then justify your answer.

[3] (i) $det\left((2A)^{-1}(A^T)(2A^T)\right) = det(A)$ for all 2006×2006 invertible matrices A.

[3] (ii) A product of elementary matrices may be singular (*i.e.*, non-invertible).

[3] (iii) Let $A = [a_{ij}]$ be a 2006×2006 matrix such such that

$$a_{ij} = \begin{cases} 1, & \text{if } i \leq j, \\ 0, & \text{if } i > j. \end{cases}$$

In other words, all entries below the main diagonal of A are equal to zero, and all other entries are equal to 1. Then A is singular (*i.e.*, non-invertibe).

[10] 4. Let A be a 2×2 matrix. Suppose that the matrix B is obtained from A, and the matrix I is obtained from B by the following elementary row operations:

$$A \xrightarrow{\text{add 3 times row 2 to row 1}} B \xrightarrow{\text{multiply row 2 by (-4)}} I$$

[5] (i) Find elementary matrices E_1 and E_2 such that $E_1A = B$ and $E_2E_1A = I$

[5] (ii) Express A as a product of elementary matrices

[6] 5. Suppose that $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$.

[3] (i) Find A^{-1}.

[3] (ii) Let $C = \begin{bmatrix} 1 & -1 \\ 0 & 4 \end{bmatrix}$. Find a matrix B such that $AB = C$, and explain why your answer is the only one possible.

[9] 6. Let A be a 4×4 invertible matrix and let

$$B = adj(A) = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 4 & 3 & 2 \\ 0 & -2 & -1 & 2 \end{bmatrix}$$

[4] (i) Calculate $det(B)$.

[5] (ii) Determine the value of $det(A)$.

[6] 7. Suppose that A is a 6×6 matrix such that $A^2 + I = 0$. Find all possible values of $det(A)$.

[4] Bonus. **WARNING: This is an <u>optional</u> BONUS question. Attempt it only if you have enough time and have solved all other problems.**

Let W be a 2006×2006 matrix such that $(W - I)^T = 2W$. Prove that W is symmetric.