Identify your section by marking an X in the box.

<table>
<thead>
<tr>
<th>Section</th>
<th>Instructor</th>
<th>Slot</th>
<th>Time</th>
<th>Room</th>
</tr>
</thead>
<tbody>
<tr>
<td>A01</td>
<td>E. Schippers</td>
<td>5</td>
<td>TTh 10:00–11:15am</td>
<td>208 Armes</td>
</tr>
<tr>
<td>A02</td>
<td>N. Zorboska</td>
<td>8</td>
<td>MWF 1:30–2:20pm</td>
<td>204 Armes</td>
</tr>
<tr>
<td>A03</td>
<td>D. Kelly</td>
<td>12</td>
<td>MWF 3:30–4:20pm</td>
<td>208 Armes</td>
</tr>
<tr>
<td>A04</td>
<td>C. Platt</td>
<td>15</td>
<td>TTh 4:00–5:15pm</td>
<td>200 Armes</td>
</tr>
<tr>
<td>A05</td>
<td>J. Sichler</td>
<td>E2</td>
<td>T 7:00–10:00pm</td>
<td>204 Armes</td>
</tr>
<tr>
<td>Other</td>
<td>(challenge, deferred, etc.)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Instructions

Fill in all the information above.

This is a two-hour exam.

No calculators, texts, notes, or other aids are permitted.

Show your work clearly for full marks.

This exam has 11 questions on 7 numbered pages, for a total of 120 points. Check now that you have a complete exam.

Answer all questions on the exam paper in the space provided. If you need more room, you may continue your answer on the reverse side, but clearly indicate that your work is continued there. There are two blank pages at the end for scratch work, but nothing on these pages will be marked. You may also use the backs of numbered pages for scratch work, but none of it will be marked unless clearly indicated otherwise.

Do not separate any pages.
1. Let \( A = \begin{bmatrix} 1 & 2 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 \\ 2 & 4 & 0 & 2 & 1 \end{bmatrix} \) and \( b = \begin{bmatrix} 2 \\ 6 \\ 5 \end{bmatrix} \).

(a) Find the reduced row echelon form of the augmented matrix \([ A \mid b ]\).

(b) Find the general solution of the system \( A x = b \), entering your answer in the spaces provided:

\[ x_1 = \quad \]

\[ x_2 = \quad \]

\[ x_3 = \quad \]

\[ x_4 = \quad \]

\[ x_5 = \quad \]
2. Let \( A = \begin{bmatrix} 2 & 4 & 1 & 1 \\ 2 & 4 & 1 & 2 \\ 2 & 4 & 2 & 3 \\ 2 & 5 & 0 & 1 \end{bmatrix} \). Find \( \det(A) \) by first reducing \( A \) to an upper triangular matrix.

3. Assume \( A \) is a \( 4 \times 4 \) matrix with determinant \(-3\). Find the determinants of the inverse, \( A^{-1} \), and of the adjoint, \( \text{adj}(A) \), without calculating either \( A^{-1} \) or \( \text{adj}(A) \). Justify your answers, making sure your reasoning applies to all such matrices \( A \).

(a) \( \det(A^{-1}) = \quad \) Reason:

(b) \( \det(\text{adj}(A)) = \quad \) Reason:
4. (a) Find the area of the triangle in \( \mathbb{R}^2 \) with vertices \( P(1, 1, 1) \), \( Q(0, 2, 1) \), and \( R(2, 1, 2) \).

(b) Find the volume of the parallelepiped determined by the vectors \( u = (0, -7, 5) \), 
\( v = (2, 0, 0) \) and \( w = (1, 0, 3) \).

5. (a) Write the parametric equations of the line \( l \) in \( \mathbb{R}^2 \) that contains the points \( P(1, 0, 1) \) and \( Q(-1, 2, 1) \).

(b) Show that the line \( l \) from (a) is parallel to the plane with equation \( 2x + 2y - 7z = 4 \).

(e) Find the distance from the point \( P(1, 0, 1) \) to the plane with equation \( 2x + 2y - 7z = 4 \).
6. In $\mathbb{R}^4$, let $u = (0, 1, 0, 0)$, $v = (2, 0, k, -1)$, and $w = (-4, 0, -3, 2)$. In each part, justify your answer and if there are no such $k$, answer "none".

(a) Find all values of $k$ (if any) for which $v$ is orthogonal to $w$.

(b) Find all values of $k$ (if any) for which the set $\{u, v, w\}$ is linearly independent.

(c) Find all values of $k$ (if any) for which the set $\{u, v, w\}$ is a basis of $\mathbb{R}^4$.

7. In the polynomial space $P_2$, let $p_1(x) = 1 - x$ and $p_2(x) = x + 3x^2$.

Determine whether $p_3 = 1 + 2x + 6x^2$ is in the space spanned by $p_1$ and $p_2$, and prove your answer.
8. In each question, determine whether the given set $W$ is a subspace of the given vector space $V$. Justify your answers.

(a) $V = M_{2,2}$ and $W$ consists of all matrices of the form \[ \begin{bmatrix} a & 3 \\ 0 & 2a \end{bmatrix} \] for $a$ in $\mathbb{R}$.

(b) $V = M_{2,2}$ and $W$ consists of all matrices of the form \[ \begin{bmatrix} a & 3a \\ 0 & b \end{bmatrix} \] for $a$ and $b$ in $\mathbb{R}$.

(c) Let $a = (2, 0, -1)$. $V = \mathbb{R}^3$ and $W$ is the set of all vectors $u = (x, y, z)$ such that $u \times a = 0$. 

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PAGE 5

Continued
9. Let $A$ and $B$ be $n \times n$ matrices.

(a) Show that $A^2 - B^2 = (A + B)(A - B)$ if and only if $AB = BA$.

(b) If $A$ satisfies $A^2 + A - I_n = 0$, write the inverse $A^{-1}$ in terms of the matrix $A$.

10. Let $A$ be a $4 \times 6$ matrix. Answer the following questions by filling in the blanks:

(a) The largest possible dimension of the null space of $A$ is _________.

(b) The smallest possible dimension of the null space of $A$ is _________.

(c) Suppose now the rows of $A$ are linearly independent.

1. The dimension of the null space of $A$ is _________.

2. The dimension of the column space of $A$ is _________.
11. The matrix \( A = \begin{bmatrix} 1 & -2 & 3 & 0 & 2 & 0 \\ -3 & 6 & -9 & 1 & -7 & 0 \\ 2 & -4 & 6 & 0 & 4 & 0 \\ 5 & -10 & 15 & -1 & 11 & 1 \end{bmatrix} \)

has reduced row echelon form \( R = \begin{bmatrix} 1 & -2 & 3 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \).

(a) Find the dimension and a basis of the row space of \( A \).

(b) Find the dimension and a basis of the null space of \( A \).

(c) Find the dimension and a basis of the column space of \( A \).