THE UNIVERSITY OF MANITOBA

DATE: April 13, 2009

DEPARTMENT & COURSE NO. MATH 1300

EXAMINATION: Vector Geometry & Linear Algebra

PAPER NO: 97

FINAL EXAMINATION

TITLE PAGE

TIME: 2 hours

EXAMINERS: see below

NAME: (Print in ink) _____

STUDENT NUMBER: (in ink) _____

EXAMINATION ROOM _____ SEAT NO. _____

SIGNATURE: (in ink) _____

(I understand that cheating is a serious offense)

Please indicate your instructor and section by placing a check mark in the appropriate box below.

❑	A01	**D. Kelly**	Tu, Th 10:00 am- 11:15 am
❑	A02	**M. Doob**	M, W, F 1:30 pm- 2:20 pm
❑	A03	**J. Chipalkatti**	M, W, F 3:30 pm- 4:20 pm
❑	A04	**J. Arino**	Tu, Th 4:00 pm - 5:15 pm

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. Calculators, cell phones or electronic translators are also not permitted.

This exam has a title page, 6 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but do not remove the staple.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 120.

Answer all questions on the exam paper in the space provide beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	14	
2	15	
3	24	
4	10	
5	16	
6	4	
7	18	
8	12	
9	7	
Total:	120	

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DEPARTMENT & COURSE NO. MATH 1300 | PAGE: 1 of 6

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1. (14) Let $P(-1,1,1)$, $Q(1,2,3)$ and $R(2,1,0)$ be points in R^3.

   (a) Find the equation of the plane in R^3 containing P, Q and R.

   (b) Find the equation of line L in R^3 passing through P and Q.

   (c) Find the area A of the triangle determined by P, Q and R.

   (d) Find the point of intersection of L and the xy-plane.

2. (15) Let

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Find the determinant of A.

(b) Find A^{-1}.

(c) Find *all* solutions $\mathbf{x}$ to

$$A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

(d) Find the adjoint of A.

3. (24) Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

Evaluate the following matrices:

(a) A^T

(b) A^{-1}

(c) $(A^{-1})^T$

(d) $(A^T)^{-1}$

(e) $(A^{-1})^2$

(f) $(A^2)^{-1}$

4. (10)

(a) Calculate $\text{proj}_{\mathbf{v}}\mathbf{u}$, the projection of $\mathbf{u}$ along $\mathbf{v}$, where $\mathbf{u} = (2, -2, 3)$ and $\mathbf{v} = (2, 1, 3)$.

(b) Find a unit vector in the direction of $\mathbf{w} = (4, -3, 2, 1)$.

(c) Find all values t so that vectors $(2, 5, -3, 6)$ and $(4, t, 7, 1)$ are orthogonal.

5. (16) Let $\mathbf{u} = (1, 0, 1)$ and $\mathbf{v} = (0, 1, 1)$.

   (a) Find a vector $\mathbf{w}$ in R^3 that is *not* in the span of $\{\mathbf{u}, \mathbf{v}\}$. You *must* justify your answer.

   (b) Find a vector $\mathbf{w}$ in R^3 so that $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a basis for R^3. You *must* justify your answer.

   (c) Let $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ be the basis you gave in part (b). Find real numbers a, b and c so that $(25, 13, -17) = a\mathbf{u} + b\mathbf{v} + c\mathbf{w}$.

   (d) Show that $C = \begin{bmatrix} 11 & 17 \\ 1 & 13 \end{bmatrix}$ is in the span of $\{A, B\}$ where $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 1 \\ 3 & 4 \end{bmatrix}$.

6. (4) In this question, each answer is a number.

   (a) The smallest possible value for the dimension of the null space of a 17×23 matrix is _____.

   (b) The largest possible value of the dimension of the column space of a 17×23 matrix is _____.

   (c) Let A be a 17×23 matrix whose null space has dimension 15. The dimension of the row space is _____.

   (d) the span of $(1, 1, 1, 1, 1)$, $(1, 1, 0, 0, 1)$ and $(0, 0, 1, 1, -1)$ has dimension _____.

7. (18) The matrix

$$A = \begin{bmatrix} 2 & -10 & 1 & 1 & 22 & -2 & 0 \\ 4 & -20 & 1 & 1 & 36 & 0 & 16 \\ 1 & -5 & 2 & 2 & 23 & 3 & 26 \\ 4 & -20 & -2 & -2 & 12 & 3 & 19 \end{bmatrix}$$

has reduced row echelon form

$$R = \begin{bmatrix} 1 & -5 & 0 & 0 & 7 & 0 & 3 \\ 0 & 0 & 1 & 1 & 8 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for the null space of A.

(b) Find a basis for the row space of A.

(c) Find a basis for the column space of A.

(d) The dimension of the null space of A is _____

(e) The dimension of the row space of A is _____

(f) The dimension of the column space of A is _____

8. (12) In each part of this question a *subset* W of the vector space R^5 is defined. State whether or not W is a subspace of R^5. *Justify* your answer.

   (a) $W = \{(0, a, 2a+3b, 0, 5a-6b+3c) \mid a, b, c \text{ in } \mathrm{R}\}$.

   (b) $W = \{(0, a, 2a+3, 0, a+b+c) \mid a, b, c \text{ in } \mathrm{R}\}$.

   (c) $W = \{(a, b, c, d, e) \mid 7a+5b+3c+2d+e = 0\}$.

9. (7) Indicate whether each statement is true or false:

   (a) Whenever a finite set S spans a vector space V, then there is a subset of S which is a basis of V ..... ☐ True ☐ False

   (b) If a vector v is not in the span of $\{u_1, u_2, u_3\}$, then the set $\{u_1, u_2, u_3\}$ is linearly independent ..... ☐ True ☐ False

   (c) If V and W are subspace of some R^n, then the set $\{v+w \mid v \text{ in } V \text{ and } w \text{ in } W\}$ is a subspace of R^n ..... ☐ True ☐ False

   (d) There is a basis for R^6 which contains the following three vectors: $(5,3,4,7,5,6)$, $(0,6,5,4,3,7)$ and $(0,0,9,2,10,3)$ ..... ☐ True ☐ False

   (e) $\det(-A) = -\det(A)$ ..... ☐ True ☐ False

   (f) $\det(AB) = \det(A)\det(B)$ ..... ☐ True ☐ False

   (g) $\det(A+B) = \det(A) + \det(B)$ ..... ☐ True ☐ False