The University of Manitoba

Vector Geometry and Linear Algebra (MATH 1300)
Midterm Examination: Winter 2009
February 24, 2009.

Time: One hour
Total Marks: 60

Last Name (CAPITAL LETTERS ONLY): ________________________________

Student Number: ________________________________

First Name (CAPITAL LETTERS ONLY): ________________________________

Signature: ________________________________

(I acknowledge that cheating is an extremely serious offense.)

Place a check mark (√) in the box corresponding to your section.

☐ D. Kelly A01 T-Th 10-11:15
☐ M. Doob A02 M-W-F 1:30-2:20
☐ J. Chipalkatti A03 M-W-F 3:30-4:20
☐ J. Arino A04 T-Th 4:00-5:15

Instructions:
Please ensure that your paper has a total of 4 pages (including this page). Read the questions thoroughly and carefully before attempting them. You must show your work in reasonable detail to get credit.

You are not allowed to use any of the following: calculators, notes, books, dictionaries or electronic communication devices (e.g., cellular phones, pagers or blackberries). You may use the left-hand pages for rough work.

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Q1. Let $A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 3 & -1 \end{bmatrix}$. Find the inverse of $A$ using elementary row operations. (Hint: there are no fractions in $A^{-1}$.)

Q2. Let $A, B$ and $C$ be matrices such that $A$ has size $5 \times 7$. Assume that the matrix $AB^{-1} + C^T B$ is defined. What are the sizes of $B$ and $C$?

Answers: The size of $B$ is ________ The size of $C$ is ________
Q3. Let \( A = \begin{bmatrix} -4 & 3 \\ 1 & 0 \end{bmatrix} \). Find \( 2 \times 2 \) elementary matrices \( E_1, E_2 \) and \( E_3 \) such that \( A = E_1 E_2 E_3 \). [10]

Q4. Assume that the matrix
\[
\begin{bmatrix}
1 & a - 2b + 7 & b + 2 & 2a - 6 \\
0 & 1 & a + 4b & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
is in reduced row-echelon form. Find the values of \( a \) and \( b \).

Answers: \( a = \) _____ \( b = \) _____

Q5. Let \( A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & x - 1 \\ 0 & 2 & -5 \end{bmatrix} \). If \( \det(-2A) = 88 \), then find the value of \( x \). [8]

Answer: \( x = \) _____
Q6. Consider the following system of linear equations:

\[ \begin{align*}
2x + y &= 1, \\
x - 4y &= 14.
\end{align*} \]

Find the values of \( x \) and \( y \) using Cramer's rule. No credit will be given for any other method.

Answers: \( x = \quad y = \quad \)

Q7. Let \( A = \begin{bmatrix} -3 & 2 & 1 \\ 4 & 7 & -2 \\ 1 & 3 & -1 \end{bmatrix} \). Find the entry in the 3rd row and 2nd column of \( A^{-1} \).

(Hint: use the formula for \( A^{-1} \) in terms of \( \text{adj}(A) \).)

Answer: The entry is = ______