**THE UNIVERSITY OF MANITOBA**

DATE: October 28, 2005 — MIDTERM EXAMINATION
TITLE PAGE
DEPARTMENT & COURSE NO: 136.150 — TIME: 1 hour
EXAMINATION: Introductory Calculus — EXAMINER: Staff

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FAMILY NAME: (Print in ink) _____

FIRST NAME: (Print in ink): _____

STUDENT NUMBER: _____

SIGNATURE: (in ink) _____
(I understand that cheating is a serious offense)

**Please mark your section number.**

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|---|---|---|---|---|---|
| ☐ | Section | L01 | Slot 3,5T | M, W,F, 10:30 & T 10:00AM | **P. Penner** |
| ☐ | Section | L02 | Slot 2 | M,W,F, 9:30 AM | **P. N. Shivakumar** |
| ☐ | Section | L03 | Slot 5 | T & Th, 10:00 AM | **S. Kalajdzievski** |
| ☐ | Section | L04 | Slot 6 | M,W,F, 11:30 AM | **W. Korytowski** |
| ☐ | Section | L05 | Slot 7 | M,W,F, 12:30 PM | **A. Gumel** |
| ☐ | Section | L06 | Slot 12 | M,W,F, 3:30 PM | **M. Young** |
| ☐ | Section | L07 | Slot E2 | T, 7:00 PM | **J. Sichler** |

**INSTRUCTIONS TO STUDENTS:**

This is a 1 hour exam. **Please show your work clearly.**

No calculators, texts, notes, cell phones, translators or other aids are permitted.

This exam has a title page, 5 pages of questions and 1 blank page at the end for rough work. Please check that you have all the pages.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 60.

**Answer all questions on the exam paper** in the space provide beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

| **DO NOT WRITE IN THIS COLUMN** |
|---|
| 1. _____ /11 |
| 2. _____ /10 |
| 3. _____ /14 |
| 4. _____ /6 |
| 5. _____ /7 |
| 6. _____ /4 |
| 7. _____ /8 |
| **TOTAL** _____ /60 |

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October 28, 2005 — MIDTERM EXAMINATION
DEPARTMENT & COURSE NO:136.150 — PAGE NO: 1 of 5
EXAMINATION: Calculus — TIME: 1 HOURS — EXAMINER: ( Staff )

**Values**

[11] 1. Find each limit, if it exists. If the limit does not exist, indicate whether it tends to $\infty$ or $-\infty$, or neither .

(a) $\lim_{x\to 3}\left(\frac{1}{x-3}-\frac{6}{x^2-9}\right)$

(b) $\lim_{x\to 1}\frac{x^2-4x+3}{x^2-1}$

(c) $\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$

**Values**

[10] 2. Find the constants $a$ and $b$ such that

$$f(x)=\begin{cases} 2 & x\le -1 \\ ax+b & -1<x<3 \\ -2 & x\ge 3 \end{cases}$$

is continuous everywhere. Use limits to justify your answers

**Values**

[14] 3. Find $f'(x)$. DO NOT SIMPLIFY YOUR ANSWERS.

(a) $f(x) = 4\sqrt[3]{x} + \sec x + \frac{1}{x^2} + \sin 2$

(b) $f(x) = \frac{x}{x^2 + \cos x}$

(c) $f(x) = e^{x \tan x}$

[6] 4. Using the definition of the derivative find $f'(x)$ if $f(x) = \frac{1}{x+3}$

**Values**

[7] 5. Find an equation of the tangent line, at the origin, to the curve described by $2e^{-x} + e^{y} = 3e^{x-y}$.

[4] 6. Prove the following theorem (using the definition of derivative):
If $f$ and $g$ are functions differentiable for all real numbers then

$$(f+g)'(x) = f'(x) + g'(x)$$

**Values**

[8] 7. A conical paper cup with a diameter of 8 cm across the top and 6 cm deep is full of water. The cup springs a leak at the very bottom tip and loses water at the rate of 2 $cm^3$ per minute. How fast is the water level dropping at the instant when the water is exactly 3 cm deep?

( Recall that the volume of a cone with radius $r$ and height $h$ is $V = \frac{\pi r^2 h}{3}$.)