UNIVERSITY OF MANITOBA

DATE: December 14, 2006
PAPER # 406
DEPARTMENT & COURSE NO: MATH 1500
EXAMINATION: Intro Calculus

FINAL EXAMINATION
TITLE PAGE
TIME: 2 hours
EXAMINER: Various

SURNAME NAME: (Print in ink) _____

FIRST NAME: (Print in ink) _____

STUDENT NUMBER: _____ SEAT NUMBER: _____

SIGNATURE: (in ink) _____

(I understand that cheating is a serious offense)

Please indicate your instructor and section by checking the appropriate box below:

☐	A01	slot 3,5T	MWF -10-30 and T - 10:00	P. Penner
☐	A02	slot 2	MWF-9:30	A. Gerhard
☐	A03	slot 5	T,Th-10:00	C. K. Gupta
☐	A04	slot 6	MWF-11:30	W. Korytowski
☐	A05	slot 7	MWF-12:30	P.N. Shivakumar
☐	A06	slot 12	MWF-3:30	M. Young
☐	A07	E2	Tu 7:00	J. Sichler
☐	A91	Challenge for Credit	☐ Dakota ☐ Sisler	

☐ Deferred exam

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 7 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank page if you want, but do not remove the staple.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 120 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	25	
2	10	
3	30	
4	7	
5	7	
6	13	
7	4	
8	12	
9	12	
Total:	120	

[25] 1. Find $\frac{dy}{dx}$ for the following **(DO NOT SIMPLIFY)**:

(a) $y = e^{5x}\ln(2 - x^3)$

(b) $y = \frac{x^2}{2^x}$

(c) $y = (\ln x)^{(x^2)}$

(d) $x^y = y^x$

[10] 2. Prove the following: If $f'(x) = 0$ for all x in an interval I, then f is constant on I.

[30] 3. If $f(x) = \dfrac{8(x-2)}{x^2}$ then $f'(x) = \dfrac{-8(x-4)}{x^3}$ and $f''(x) = \dfrac{16(x-6)}{x^4}$.

(a) Find all intercepts of the function; state the domain.

(b) Calculate all limits associated with any horizontal and vertical asymptotes to the curve $y = f(x)$. Also, give the equations of these asymptotes, if any.

(c) Find the critical points of $f(x)$, the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing. Find the coordinates of any local maxima and/or minima of $f(x)$.

(d) Find where the $f(x)$ is concave up, where $f(x)$ is concave down. Find inflection points if any.

(e) Sketch the curve $y = f(x)$, displaying the information found in parts (a),(b), (c) and, (d).

For the following two questions it is sufficient to write down a function and indicate an interval on which it is defined. The function and interval should be chosen so that in order to solve the problem you would find the maximum or minimum value of the function on that interval. **IT IS NOT NECESSARY TO ACTUALLY FIND THE MAXIMUM OR MINIMUM VALUE.**

[7] 4. Two equal adjacent rectangular areas are to be created with fencing. (The two areas are to be separated with one line of fencing.) If 100 m of fence is available, what is the largest total area which can be fenced?

[7] 5. A piece of wire 10 cm long is to be cut into two pieces. One piece is bent into an equilateral triangle and the other into a square. How should the wire be cut to maximize the area? You are allowed to use all the wire for one shape and none of the wire for another (that is to not cut the wire). (The area of an equilateral triangle is $\frac{\sqrt{3}}{4}s^2$ where s is the length of a side.)

[13] 6. Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 + 3x^2 - 12x + 1$ on the interval $[-1, 2]$.

[4] 7. Let $f(x) = \int_0^x (e^{\cos t} - \tan t)\, dt$ for $-\pi/2 < x < \pi/2$. Find $f'(0)$.

[12] 8. Find the total area enclosed by the curve $y = x(x-1)(x+1)$ and the x-axis.

[12] 9. Evaluate the following integrals:

(a) $\int (x^4 - \frac{1}{x^2} + e^x + \sec^2(2x))\, dx$

(b) $\int_{-1}^{1} (x^3 - x)\, dx$