INSTRUCTIONS TO STUDENTS:

This is a 1 hour exam. Please show your work clearly.

No texts, notes, or other aids are permitted. Calculators, cellphones or electronic translators are also not permitted.

This exam has a title page, 5 pages of questions and also 1 blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but do not remove the staple.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 60 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

<table>
<thead>
<tr>
<th>Question</th>
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<td><strong>Total:</strong></td>
<td><strong>60</strong></td>
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1. Evaluate the following limits. If the limit does not exist or is ±∞ indicate that.

(a) \[ \lim_{x \to 4} \frac{x^2 - 16}{x^2 - 4x} \]

(b) \[ \lim_{x \to 3} \frac{x^2 + 9}{x^2 - 9} \]

(c) \[ \lim_{x \to 0} \frac{x}{\sqrt{x} + 1 - 1} \]
2. Find the derivative \( f'(x) \) in each case. DO NOT SIMPLIFY your answers.

(a) \[ f(x) = 2\sqrt{x} + \frac{5}{x^3} + \pi^2 \]

(b) \[ f(x) = x^2 \cos x \]

(c) \[ f(x) = \frac{x^2}{\tan(3x)} \]

(d) \[ f(x) = \sin^2(\sqrt{x^2 + 1}) \]
3. Let

\[ f(x) = \begin{cases} 
\sqrt{x^2 + 1} & \text{if } x > 0; \\
x^2 + 1 & \text{if } x < 0; \\
2 & \text{if } x = 0.
\end{cases} \]

Does \( \lim_{x \to 0} f(x) \) exist? If so what is its value? Is the function continuous at \( x = 0 \)? You MUST justify your answers to receive full marks.

4. Prove that \( (\sin x)' = \cos x \).

You may assume the trigonometric identity \( \sin(a + b) = \sin a \cos b + \cos a \sin b \) and the two limits \( \lim_{h \to 0} \frac{\sin h}{h} = 1 \) and \( \lim_{h \to 0} \frac{\cos h - 1}{h} = 0 \).
5. Let \( y \) be a function of \( x \) which satisfies the equation \( y \sin x = x^3 + \cos y \). Find \( y' \).

6. Use only the definition of the derivative to find \( f'(x) \) if \( f(x) = \frac{1}{x^2 + 1} \).
7. A line is tangent to the curve $y = x^2 + 2x - 1$ when $x = 0$. Find where this line intersects the $x$-axis.

8. The length and width of a rectangle are changing but the area remains fixed at 10 cm$^2$. If the length is increasing at 3 cm per minute when it is 5 cm long, how fast is the width changing at that time?