UNIVERSITY OF MANITOBA

DATE: December 13, 2007 | FINAL EXAMINATION
PAPER # 388 | TITLE PAGE
DEPARTMENT & COURSE NO: MATH 1500 | TIME: 2 hours
EXAMINATION: Intro Calculus | EXAMINER: Various

SURNAME NAME: (Print in ink) _____

FIRST NAME: (Print in ink) _____

STUDENT NUMBER: (in ink) _____

EXAMINATION ROOM: _____ SEAT NUMBER: _____

SIGNATURE: (in ink) _____

(I understand that cheating is a serious offense)

Please indicate your instructor and section by checking the appropriate box below:

☐	A01	slot 3,5T	MWF -10-30 and T - 10:00	D. Kalajdzievska
☐	A02	slot 2	MWF-9:30	M. Young
☐	A03	slot 5	T,Th-10:00	C. K. Gupta
☐	A04	slot 6	MWF-11:30	A. Gerhard
☐	A05	slot 7	MWF-12:30	F. Ghahramani
☐	A06	slot 12	MWF-3:30	P.N. Shivakumar
☐	A07	E2	Tu 7:00	J. Sichler
☐	A91	Challenge for Credit	☐ Dakota ☐ Sisler	

☐ Deferred exam

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 7 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank page if you want, but do not remove the staple.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 120 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	30	
2	12	
3	30	
4	15	
5	14	
6	12	
7	7	
Total:	120	

[30] 1. Find $\frac{dy}{dx}$ for the following **(DO NOT SIMPLIFY)**:

(a) $y = \frac{2x-3}{x^2}$

(b) $y = \log_3(2^{\sin x})$

(c) $y = \left(\frac{3}{x}\right)^x$

(d) $y = \int_0^{x^2} \frac{\sin t}{t+2}\, dt$

(e) $x^3 - xe^y = y$, when $x = 1$, $y = 0$

[12] 2. Prove the following: If $f'(x) > 0$ for all x in an interval I, then f is increasing on I.

[30] 3. If $f(x) = \dfrac{(x^2-1)}{(x-2)^2}$ then $f'(x) = \dfrac{-2(2x-1)}{(x-2)^3}$ and $f''(x) = \dfrac{2(4x+1)}{(x-2)^4}$.

(a) State the domain of the function.

(b) Find all intercepts of the function.

(c) Calculate all limits associated with any horizontal and vertical asymptotes to the curve $y = f(x)$. Also, give the equations of these asymptotes, if any.

(d) Find the critical points of $f(x)$, the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing. Find the coordinates of any local maxima and/or minima of $f(x)$.

(e) Find where $f(x)$ is concave up, where $f(x)$ is concave down. Find the coordinates of any inflection points.

(f) Using the information obtained in parts (a)–(e) above, sketch the graph of the function $y = f(x)$.

[15] 4. At 1 pm, ship A is 5 km due south of Ship B. Ship A sails north at 3 km/hr and ship B sails west at 4 km/hr. When is the distance between the ships a minimum? How far are they apart at that time?

[14] 5. Find the absolute maximum and absolute minimum values of the function $f(x) = x^4 - 2x^2 + 7$ on the interval $[-1, 2]$.

[12] 6. (a) Express the total area bounded by the curve $y = \cos x$, the lines $x = 0$ and $x = \pi$ and the x-axis as an integral or integrals. Find the area by evaluating the integral(s).

(b) Evaluate $\int_0^2 \sqrt{4 - x^2}\, dx$ by recognizing the area which it represents and finding that area.

[7] 7. Find the most general antiderivative of $\frac{(\sqrt{x}+1)^2}{x}$, that is evaluate $\int \frac{(\sqrt{x}+1)^2}{x}\,dx$.