UNIVERSITY OF MANITOBA

DATE: October 29, 2009 5:30 PM

MIDTERM EXAM
6 PAGES
TIME: 1 hour
EXAMINER: Kucera et al

DEPARTMENT & COURSE NO: MATH 1500
EXAMINATION: Calculus I

NAME: (Print in ink) _____

STUDENT NUMBER: _____

SIGNATURE: (in ink) _____
(I understand that cheating is a serious offense)

- ❑ A01 D. Kaladjievska
- ❑ A02 E. Schippers
- ❑ A03 R. Craigen
- ❑ A04 T. Kucera
- ❑ A05 T. Mohammed
- ❑ A06 C. Podder
- ❑ A07 J. Sichler
- ❑ Challenge for credit
  - ❑ Dakota
  - ❑ Sisler

INSTRUCTIONS TO STUDENTS:

This is a 1 hour exam. **Please show your work clearly.**

No calculators, cellphones, electronic translators, texts, notes or other aids, are permitted.

This exam has a title page, 6 pages of questions and also one blank page for rough work. Please check that you have them all. You may remove the blank page if you want, but be careful not to loosen the staple.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 60 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	6	
2	16	
3	10	
4	6	
5	16	
6	6	
Total:	60	

SOME LIMIT FORMULAS YOU MAY USE WITHOUT PROOF

$$\lim_{h\to 0}\frac{\sin h}{h}=1 \qquad \lim_{h\to 0}\frac{\cos h-1}{h}=0 \qquad \lim_{h\to 0}\frac{e^h-1}{h}=1$$

[6] 1. Let

$$g(x) = \begin{cases} x + b & x < 0 \\ \cos x & x \geq 0 \end{cases}$$

(a) Is there a value of b that makes this function continuous at $x = 0$? (Give reasons for your answer.)

(b) Is there a value of b that makes this function differentiable at $x = 0$? (Give reasons for your answer.)

2. Find the following limits if they exist. If a limit does not exist indicate why not and whether the function tends to ∞ or $-\infty$

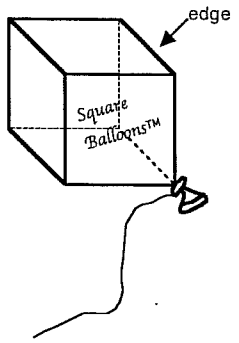
[4] (a) $\lim_{x \to 3} \frac{x^2 - 3x}{x^2 - 9}$

[4] (b) $\lim_{x \to 5} \frac{\sqrt{x-1} - 2}{5 - x}$

[4] (c) $\lim_{x \to 2^-} \frac{5}{\sqrt{2 - x}}$

[4] (d) $\lim_{x \to -\infty} \frac{x^3 - x - 1}{x^3 - |x^3| + 1}$

[10] 3. A novelty toy company is testing a prototype of a balloon they have just invented, whose shape remains a perfect cube (see diagram) when it is inflated to any size.



(a) Express the volume of the balloon as a function of the edge length.

(b) At a certain time the edge length of the balloon is 50 cm and the volume is increasing at a rate of 15000 cm^3/sec. At that time, how fast is the edge length increasing?

(c) Express the surface area of the balloon as a function of the edge length.

(d) Find the rate of change of the surface area at the time described in part (b) of this question.

[6] 4. Find the equation of the tangent line to the graph of

$$x^3y^3 - y = 2x$$

at the point $(0, 0)$.

5. Differentiate each of the following functions with respect to x. Do not simplify.

[4] (a) $y = x^3 + \dfrac{3}{\sqrt[3]{x}} + e^x - x^e - \pi^2$.

[4] (b) $y = \tan(\sin(1 + x^2))$

[4] (c) $y = (x^2 - e^x)\left(\cot x - \dfrac{1}{x^2}\right)$

[4] (d) $y = \dfrac{\sqrt[4]{x}}{5x + 1}$

[6] 6. Prove the required theorem:

$$\frac{d}{dx}\sin x = \cos x.$$