UNIVERSITY OF MANITOBA

DATE: February 29, 2008

DEPARTMENT & COURSE NO: MATH 1500
EXAMINATION: Intro Calculus

NAME: (Print in ink) _________________________________
STUDENT NUMBER: (in ink) ____________________________
SIGNATURE: (in ink) ________________________________
(I understand that cheating is a serious offense)

Please indicate your instructor and section by checking the appropriate box below:

☐ A01 slot 3,5T    MWF 10-30 and T 10:00    W. Korytowski
☐ A02 slot 1      MWF 8:30                   A. Gerhard
☐ A03 slot 4      T, Th 8:30                 W. Korytowski
☐ A04 slot 9      T, Th 11:30                 T. Kucera
☐ A05 slot 15     T, Th 4:00                  D. Kalajdzievska
☐ A06 slot 10     T, Th 1:00                  C. K. Gupta
☐ A91 Challenge for Credit    ☐ SJR

INSTRUCTIONS TO STUDENTS:

This is a 1 hour exam. Please show your work clearly.

No texts, notes, or other aids are permitted. Calculators, cellphones or electronic translators are also not permitted.

This exam has a title page, 5 pages of questions and also 1 blank page for rough work. Please check that you have all the pages. You may remove the blank page if you want, but do not remove the staple.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 60 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.
1. Evaluate the following limits. If the limit does not exist or is $\pm\infty$ indicate that.

\[\text{[4] } \lim_{x\to 2} \frac{\sqrt{x+2} - \sqrt{2x}}{x-2}\]

\[\text{[3] } \lim_{x\to -\infty} \frac{\sqrt{x^2 + 2x}}{2x + 1}\]

\[\text{[4] } \lim_{x\to -2^-} \frac{x^2 + 5x + 6}{x(x + 2)^2}\]
2. Find the derivative $f'(x)$ in each case. DO NOT SIMPLIFY your answers.

\[ \text{[4]} \quad (a) \quad f(x) = (5x)^5 + \frac{1}{5x} + (5x)^{1/5} + (5\pi)^{-5} \]

\[ \text{[4]} \quad (b) \quad f(x) = (x^2 + 1) \tan x \]

\[ \text{[4]} \quad (c) \quad f(x) = \frac{1 + \cos x}{1 + \sin x} \]

\[ \text{[4]} \quad (d) \quad f(x) = e^{\sqrt{x^2 + 1}} \]
3. Let

\[ f(x) = \begin{cases} 
  x^2 + x & \text{if } x \geq -1; \\
  k^2 + kx & \text{if } x < -1.
\end{cases} \]

Find the value or values of \( k \) for which \( f \) is continuous at \( x = -1 \). You MUST use limits to justify your answers.

4. If \( f \) is differentiable, prove that

\[ \frac{d}{dx} [cf(x)] = c \frac{d}{dx} f(x). \]
5. Let $y$ be a function of $x$ which satisfies the equation $2x + y - \sqrt{2}\sin(xy) = \frac{\pi}{2}$.

Find the slope of the tangent line to the function at the point $\left(\frac{\pi}{4}, 1\right)$.

6. Use only the definition of the derivative to find $f'(x)$ if $f(x) = \frac{1}{2x + 1}$. 
7. A light sits on the ground 20m from a building. A man 2m tall walks away from the light directly toward the building at 1m/s. How fast is the length of his shadow on the building changing when he is 14m from the building?