THE UNIVERSITY OF MANITOBA

DATE: December 6, 2008
DEPARTMENT & COURSE NO. MATH 1700
EXAMINATION: Calculus 2
PAPER NO: 90

FIRST NAME: (Print in ink) ____________________________
FAMILY NAME: (Print in ink) ____________________________
STUDENT NUMBER: (in ink) ____________________________
EXAMINATION ROOM ________________________ SEAT NO. _______
SIGNATURE: (in ink) ____________________________
(I understand that cheating is a serious offense)

Please indicate your instructor and section by placing a check mark in the appropriate box below.

☐ A01 S. Kalajdzievski M, W, F 8:30 am- 9:20 am
☐ A02 W. Korytowski T, Th 1:00 pm – 2:15 pm
☐ Deferred Exam

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. Calculators, cell phones or electronic translators are also not permitted.

This exam has a title page, 8 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but do not remove the staple.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 120.

**Answer all questions on the exam paper** in the space provide beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

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1. Consider the curve \( x = \frac{t^3}{3} - \frac{t^3}{2}, y = e^t - t \).

(a) Find the equation of one vertical tangent line to this curve.

(b) Compute \( \lim_{t \to 0} \frac{dy}{dx} \).

2. Sketch the region \( R \) bounded by the curve \( x = -y^2 + 2 \) and the line \( y = x \).
(Exercise 2 continuing ...)

[6] (b) Find the area of $R$.

[6] (c) Set up, but DO NOT EVALUATE the integral for the volume of the solid obtained by rotating the region $R$ around the line $y = -2$. 
3. (a) Sketch the curves $r = 2$ and $r = 4 \cos \theta$ given in polar coordinates.

(b) Set up but DO NOT EVALUATE the integral for the area of the region inside both of the curves $r = 2$ and $r = 4 \cos \theta$. 
4. (a) Write the general form (in terms of unknown coefficients) of the partial fractions decomposition (expansion) of each of the following expressions. DO NOT SOLVE FOR THE COEFFICIENTS.

\[
\frac{1}{(x - 2)(x + 1)^2(x^2 + 3)^2} =
\]

(b) Evaluate \[ \int \frac{x}{x^2 - 5x + 6} \, dx \]
5. Evaluate the following integrals.

(a) \( \int \frac{1}{\sqrt{1 - (x + 1)^2}} \, dx \)

(b) \( \int x \ln(x + 1) \, dx \)

(c) \( \int \sqrt{4 - x^2} \, dx \)
(Exercise 5, continuing …)

\[ \frac{\pi}{2} \]

\[ 9 \]

(d) \( \int_0^\pi (1 - \cos x) \sin^3 x \, dx \)


[8] (a) Evaluate the following improper integral, or show it diverges.

\[ \int_0^\infty \frac{dx}{x (\sqrt{x} - 1)} \]
Exercise 6, continuing ...

[7] (b) Determine if the following improper integral converges or if it diverges. (Do not try to evaluate the given integral directly.) \[ \int_{3}^{\infty} \frac{dx}{x - (1 + \sin x)} . \]

[10] 7. Find the arc length of the arc cut from the curve \( y = x^{\frac{3}{2}} \) by the line \( y = x \).
8. Set up the following integrals. DO NOT EVALUATE the integrals.

(a) The integral for the arc length of the cardioid \( r = 3 + 3\cos\theta \) given in polar coordinates.

(b) Consider the part of the curve \( y = (2x - 4)(4 - x) \) that is above the \( x \)-axis. Set up, but DO NOT EVALUATE, the integral for the surface area of the surface obtained by rotating that part of the curve around the \( x \)-axis.