THE UNIVERSITY OF MANITOBA

DATE: Dec. 14 2005 FINAL EXAMINATION

DEPARTMENT & COURSE NO: 136.272 TIME: 2 hours

PAPER NO.: 292

EXAMINATION: Calculus 3A EXAMINER: Sasho Kalajdzievski

NAME: (PRINT)_____

STUDENT NUMBER:_____

SEAT NUMBER:_____

SIGNATURE:_____
(I understand that cheating is a serious offense)

INSTRUCTIONS TO THE STUDENT

This is a two-hours exam. There are 8 pages of questions and one blank page for rough work. Check now that you have all 8 pages of questions.. Answer all the questions in the spaces provided. If necessary, you may continue your work on the reverse sides of the pages but PLEASE INDICATE CLEARLY that your work continues and where the continuation may be found. DO NOT detach any question pages from the exam.

The point value of each question is indicated to the left of the question number. The maximum score possible is 120 points.

Please present your work CLEARLY and LEGIBLY, and use a pen (not red) or a dark pencil. **Justify your answers unless otherwise stated.**

NO CALCULATORS, TEXTS, NOTES OR OTHER AIDS ARE PERMITTED.

Values

1.

[7] **(a)** Show that the angle between the tangent vector to the curve $\mathbf{r}(t)=\left(t,\frac{t^2}{3},\frac{2t^3}{27}\right)$ at any point and the vector $\mathbf{a}=(1,0,1)$ is always a constant. Find that constant.

[7] **(b)** Find the curvature κ of $\mathbf{r}(t)=\left(t,\frac{t^2}{3},\frac{2t^3}{27}\right)$ at the point $(3,3,2)$.

Values

2. Evaluate the following limits or show they do not exist:

[4] **(a)** $\lim_{(x,y)\to(1,1)} \dfrac{x^2+y^2-1}{x^2-y^2+1}$

[6] **(b)** $\lim_{(x,y)\to(0,0)} \dfrac{2x}{x^2+x+y^2}$

[6] **(c)** $\lim_{(x,y)\to(0,0)} \dfrac{\ln(x^2+y^2+1)}{x^2+y^2}$

3.

[7] **(a)** Find the equation of the tangent plane to the surface $f(x,y)=\sqrt{2xy}$ at the point $(2,1,2)$.

Values

[7] **(b)** Consider the level surface $2xy - z^2 = 0$ and the level surface $x^2 + y^2 + z^2 = 1$. Show that they are orthogonal at their points of intersection. (Recall that two surfaces are orthogonal if their normal lines are orthogonal.)

[6] **4. (a)** Use the chain rule to find $\frac{\partial z}{\partial u}$ at the point when $u = 0$ and $v = 1$ if $z = (\sin xy) + x \sin y$, $x = u^2 + v^2$ and $y = uv$.

[6] **4 (b)** The equation $\sin(x+y) + \sin(y+z) + \sin(x+z) = 0$ defines z as a function on x and y. Find $\frac{\partial z}{\partial x}$ at the point $(\pi, \pi. \pi)$.

Values

5. Consider the function $f(x,y)=\frac{x^2}{2}+\frac{y^2}{2}$.

[4] **(a)** Find the direction in which $f(x,y)$ increases most rapidly at $(1,1)$.

[4] **(b)** Find the direction in which $f(x,y)$ decreases most rapidly at $(1,1)$.

[5] **(c)** Find the directions at which the directional derivative of $f(x,y)$ at $(1,1)$ is 0.

Values

[13] **6.** Find the absolute minimum and the absolute maximum of the function $f(x,y) = 2 + 2x + 2y - x^2 - y^2$ over the triangular domain in the first quadrant bounded by the lines $x = 0$, $y = 0$ and $y = 9 - x$.

[12] **7.** Use the method of Lagrange multipliers to find the extreme values of the function $f(x,y) = xy$ subject to the condition $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

Values

8.

[7] **(a)** Sketch the region of integration, reverse the order of integration, and evaluate

$$\int_0^1 \int_y^1 x^2 e^{xy}\, dx\, dy$$

[7] **(b)** Evaluate $\iint_R \frac{\sin x}{x}\, dx$ if R is the triangle in the xy-plane bounded by the x-axis, the line $y = x$ and the line $x = 1$.

Values

[12] **9.** Sketch and find the volume of the solid that is bounded above by the cylinder $z = x^2$ and below by the region enclosed by the parabola $y = 2 - x^2$ and the line $y = x$ in the xy-plane.