UNIVERSITY OF MANITOBA

Paper No. 579
DATE: December 21st, 2009
DEPARTMENT & COURSE NO: MATH 2720
EXAMINATION: Multiple Variable Calculus

FINAL EXAMINATION
TITLE PAGE
TIME: 2 hours
EXAMINER: Mudassar Imran

NAME: (Print in ink) _____

STUDENT NUMBER: _____

SEAT NUMBER: _____

SIGNATURE: (in ink) _____

(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 2 hours exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no cellphones or electronic translators permitted.

This exam has a title page, 10 pages of questions and also 2 blank pages for rough work. Please check that you have all the pages. You may remove the blank pages if you want, but be careful not to loosen the staples.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 100 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	8	
2	8	
3	9	
4	10	
5	10	
6	8	
7	10	
8	10	
9	12	
10	15	
Total:	100	

[8] 1. What horizontal plane is horizontal to the surface $z = x^4 - 4xy - 2y^2 + 12x - 12y - 1$ and what is the point of tangency?

[8] 2. If $z = f(x, y)$ where $x = r\cos\theta$ and $y = r\sin\theta$, then show that $\left(\frac{\partial^2 z}{\partial r^2}\right) + \frac{1}{r}\left(\frac{\partial z}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 z}{\partial \theta^2}\right) = \left(\frac{\partial^2 z}{\partial x^2}\right) + \left(\frac{\partial^2 z}{\partial y^2}\right)$

[9] 3. A hiker is standing beside a stream on the side of a mountain examining her map of the region. The height of the land (in kms) at any point (x, y) is given by $h(x, y) = \frac{20}{3+x^2+2y^2}$ where (x, y) denote the coordinates of the point on the hiker's map. The hiker is at point (3,2).

a) What is the direction of the flow of the stream at (3,2) on the hiker's map? How fast is the stream descending at her location?

b) At what angle to the path of the stream (on map) should the hiker set out if she wishes to climb at a 15° inclination to the horizontal?

c) Make a sketch of the hiker's map showing some curves of constant elevation and showing the stream.

[10] 4. Evaluate $\iint_D y dA$, where D is in the region in the first quadrant that lies above the hyperbola $xy = 1$ and below the line $y = 2$.

[10] 5. Evaluate $\iiint_E y^2 z^2 dV$, where E is bounded by the paraboloid $x = 1 - y^2 - z^2$ and the plane $x = 0$.

[8] 6. a) Evaluate $\oint_C x^2dx + y^2dy + z^2dz$, where C consists of the line segment from (0,0,0) to (1,2,-1) and from (1,2,-1) to (3,2,0).

b) Evaluate $\oint_C \mathbf{F} \cdot \mathbf{dr}$ where $\mathbf{F} = \sin x\mathbf{i} + \cos y\mathbf{j} + xz\mathbf{k}$ and $\mathbf{r} = t^3\mathbf{i} - t^2\mathbf{j} + t\mathbf{k}, 0 \leq t \leq 1$.

[10] 7. For what values of constants A and B is the vector field $\mathbf{F} = Ax\sin(\pi y)\mathbf{i} + (x^2\cos(\pi y) + Bye^{-z})\mathbf{j} + y^2e^{-z}\mathbf{k}$ is conservative. Find potential function for those A and B.

[10] 8. Use Green's theorem to evaluate the line integral along positively oriented path

$$\oint_C xe^{-2x}dx + (x^4 + 2x^2y^2)dy.$$

C is the boundary of the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

[12] 9. **Write only answers for all parts of this question**

i) Find (a) **curl** and (b) the divergence of $F(x, y, z) = (\ln x, \ln(xy), \ln(xyz))$.

ii) Is there a vector field G on $\mathcal{R}^3$ such that $\mathbf{curl}G = \langle xyz, -y^2z, yz^2\rangle$. Explain.

iii) Find the absolute max. and min. values for $f(x, y) = 4x + 6y - x^2 - y^2$ on the domain $D = \{0 \leq x \leq 4,\ 0 \leq y \leq 5\}$.

iv) Reparametrize the curve curve $r(t) = (\frac{2}{t^2+1} - 1)\mathbf{i} + \frac{2t}{t^2+1}\mathbf{j}$ with respect to arc length measured from point (1,0) in the direction of increasing t.

[15] 10. **Write only answers for all parts of this question**

a) Find the curvature $r(t) = \langle t, t^2, t^3 \rangle$ at (1,1,1).

b) Find the length of the arc of the circular helix with vector equation $r(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$ from (1,0,0) to $(1, 0, 2\pi)$.

c) Draw contour map for $f(x, y) = \frac{y}{x^2+y^2}$

d) Find the limit if it exists for $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{(x^2+y^2)}}$

e) Is the function $u = x^3 + 3xy^2$ is a solution of Laplace's equation?

f) $z = x^2 + 3xy - y^2$. If x changes from 2 to 2.05 and y changes from 3 to 2.96, find dz and ∇z.