INSTRUCTIONS TO STUDENTS:

This is a 1 hr 15 mins exam. Please show your work clearly.

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

To obtain full credit, intermediate steps in obtaining your answers must be given. Attempt all questions. If you need more space, use the back of the sheet, but if you do this indicate it clearly on the front of the sheet. There is a blank sheet at the end for you to use as scrap paper.

The total value of all questions is 40. The values of individual questions are indicated next to the statement of the questions.
1. Suppose \( z = e^r \cos \theta \), \( r = st \), \( \theta = \sqrt{s^2 + t^2} \). Use the chain rule to find \( \partial z / \partial s \) and \( \partial z / \partial t \).

2. The temperature at a point \((x, y, z)\) is given by \( T(x, y, z) = e^{-x^2 - y^2 - z^2} \). Find the rate of change of temperature at the point \( P(2, -1, 2) \) in the direction toward the point \( (3, -3, 3) \). In which direction does the temperature increase fastest at \( P \) and what is the maximum rate?
3. Let \( f(x, y) = \frac{x^2y^2 - 8x + y}{xy} \). Find all critical points of \( f \). Use the second derivative test to determine the type of each critical point. Determine whether each local extremum is also a global extremum.
[7] 4. Find the points on the surface \( x^2y^2z = 1 \) that are closest to the origin using the method of Lagrange multipliers.
5. (a) Find \( \int_0^1 \int_0^x xy^2 \, dy \, dx \)

(b) Find \( \int_0^1 \int_{2y}^1 \sin(x^2) \, dx \, dy \)

(c) Find \( \iint_R y^3 \, dx \, dy \) where \( R \) is the triangle with vertices \((0, 2), (1, 1), (3, 2)\).
6. Using Polar Coordinates, set up the integral for the volume of the solid inside the sphere \( x^2 + y^2 + z^2 = 1 \) and outside the paraboloid \( 1 - z = x^2 + y^2 \). Draw a picture. Find the value of this volume.