THE UNIVERSITY OF MANITOBA

DATE: April 17, 2008

FINAL EXAMINATION

PAPER # 330

TITLE PAGE

DEPARTMENT & COURSE NO: MATH 2720

TIME: 2 hours

EXAMINATION: Multivariable Calculus

EXAMINER: Dr. F. Ghahramani

LAST NAME : (Print in ink) _____

FIRST NAME: (Print in ink) _____

STUDENT NUMBER: (in ink) _____

SIGNATURE: (in ink) _____

(I understand that cheating is a serious offense)

INSTRUCTIONS TO CANDIDATES:

This is a 2 hour exam. **Please show your work clearly**.

No texts, notes or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 7 pages of questions and 2 blank pages for rough work. Please check that you have all the pages.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 120.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

DO NOT WRITE IN THIS COLUMN

1.	/ 20
2.	/ 15
3.	/ 15
4.	/ 10
5.	/ 10
6.	/ 20
7.	/ 20
8.	/ 10
TOTAL	/120

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Values:

1. Let C be the curve given by the vector equation

$$\vec{r}(t) = \sqrt{2}\cos t\ \vec{i} + \sin t\ \vec{j} + \sin t\ \vec{k}\ .$$

[10] a) Calculate the unit tangent vector $\overrightarrow{T(t)}$ and the unit normal vector $\overrightarrow{N(t)}$ at an arbitrary point.

[10] b) Calculate the curvature $\kappa(t)$ for the above curve in terms of t at an arbitrary point.

Values:

[15] 2. For the function

$$f(x, y) = x^3 y + x^2 - 4xy$$

determine all the critical points, and characterize at which points f has a local maximum, a local minimum or a saddle point.

Values:

3. Let $f(x, y) = x^2y + xy^2$.

[10] a) Find $\nabla f(1,1)$. In which direction does f have the greatest rate of change at the point $(1,1)$? What is the value of the greatest rate of change at this point?

[5] b) Let $\vec{u} = \frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}$. For the above function f find the directional derivative $D_u f$ at the point $(1,1)$.

Values:

[10] 4. Find the shortest distance from the point $(1, 1, 0)$ to a point on the cone $z^2 = x^2 + y^2$.

[10] 5. Let D be the closed triangular region in R^2 with vertices at the points $(0,0)$, $(0,1)$ and $(1,1)$.

Evaluate $\iint_D \cos\left(y^2\right) dA$. [**Hint:** There are two ways for writing a double integral as a repeated (iterated) integral].

Values:

[10] 6. a) Find the volume of the solid under the paraboloid $z = 4 - \left(x^2 + y^2\right)$, inside the cylinder $x^2 + y^2 = 2$ and above the xy – plane [Sketching of this solid can help. Also, you may use polar coordinates in calculating the integrals].

[10] b) Find the surface area of part of the paraboloid $z = 4 - \left(x^2 + y^2\right)$ that lies above the xy – plane.

Values:

[10] 7. a) Show that the vector field

$$\overrightarrow{F(x,y)} = \left(3x^2y + 2xy^2 + y\right)\vec{i} + \left(x^3 + 2x^2y + x\right)\vec{j}$$

is conservative and find a potential function f for it, i.e., find an f satisfying $\nabla f(x,y) = \overrightarrow{F(x,y)}$.

[10] b) For the above vector field, evaluate

$$\int_C \overrightarrow{F(x,y)}\,dr$$

where $C: \begin{cases} x = e^{\sin t} \\ y = e^{\cos t} \end{cases} \quad 0 \le t \le \dfrac{\pi}{2}$.

Values:

[10] 8. Evaluate

$$\oint_C \left(3y - e^{\sin^2 x}\right)dx + \left(8x + \sqrt{y^4 + 2}\right)dy,$$

where C is the circle $x^2 + y^2 = 4$, oriented positively and transversed once. [**Hint:** There is a clever way of doing this question using a theorem of a mathematician whose name is a colour!]