THE UNIVERSITY OF MANITOBA

DATE: April 17, 2009

PAPER NO: 398

DEPARTMENT & COURSE NO: MATH 2720

EXAMINATION: Multivariable Calculus

FINAL EXAMINATION

TITLE PAGE

TIME: 2 HOURS

EXAMINER: S. Kalajdzievski

SURNAME: (Print in ink) ________________________________

GIVEN NAME: (Print in ink) ________________________________

STUDENT NUMBER: ____________________

SEAT NUMBER ________________

SIGNATURE: (in ink) ________________

(I understand that cheating is a serious offense)

☐ A01 Tues. & Thurs 8:30 am – 9:45 am S. Kalajdzievski

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam.

Answer all questions.

Show all your work. Unless otherwise stated, answers only, without supporting work or other justification will receive no more than partial credit.

Textbooks, notes, calculators, electronic devices such as: cellular phones, translators, pagers, etc. are not permitted.

The exam consists of a title page and 7 pages of questions. The total value of all questions is 120 marks. Answer the questions in the space provided beneath the question. If additional space is required, use the back of the page to continue your answer. Indicate clearly if your answer is continued.

There is 1 blank page at the end of this booklet for rough work.

| 1. | __________ / 12 |
| 2. | __________ / 9 |
| 3. | __________ / 12 |
| 4. | __________ / 11 |
| 5. | __________ / 11 |
| 6. | __________ / 12 |
| 7. | __________ / 14 |
| 8. | __________ / 12 |
| 9. | __________ / 15 |
| 10. | __________ / 12 |

TOTAL __________ /120
1. Find the unit tangent vector $T(t)$, the unit normal vector $N(t)$ and the curvature $k(t)$ of the helix $\mathbf{r}(t) = (\cos t, \sin t, 2t)$ at the point when $t = 0$.

2. The following two questions are mutually independent. In both (a) and (b) you only need to sketch; no need to support your sketches with words. Ugly sketches will get reduced marks.

(a) Sketch in the $xy$-plane the domain of the function $f(x,y) = \frac{\sqrt{1-y}}{\sqrt{y-x^2}}$.

(b) Sketch the surface $z = x^2 + y^2 - 1$. 

3. Evaluate the following limits or show they do not exist.

(a) \[ \lim_{(x,y) \to (0,0)} \frac{xy}{2x^2 + 3y^2} \]

(b) \[ \lim_{(x,y) \to (0,0)} \frac{xy}{\sqrt{x^2 + y^2}} \] [Hint: switch to polar coordinates and use the Squeeze theorem.]

4. Denote \( F(x,y) = x^3y^3 - x^4y + 3xy - 3 \).

(a) Assume \( F(x,y) = 0 \) (i.e., \( x^3y^3 - x^4y + 3xy - 3 = 0 \)) defines \( y \) as a function of \( x \). Express \( \frac{dy}{dx} \) in terms of the partial derivatives \( \frac{\partial F}{\partial x} \) and \( \frac{\partial F}{\partial y} \), then compute its value at the point \((1,1)\).

(b) If \( x = s + t \) and \( y = st \), compute \[ \frac{\partial F}{\partial t} \]
5. (a) Find the direction in which the function \( f(x,y) = 2xy - 5y^2 \) increases fastest from the point \((2,-3)\).

(b) Find the equation of the tangent plane to the surface \( z = 2xy - 5y^2 \) at the point \((2,-3,-57)\).

6. Find and classify (using the second derivative test) all local extrema of the function \( f(x,y) = x^3 + 5x^2 + 3y^2 - 6xy \).
7. Find the absolute minimum and the absolute maximum of the function 
\[ f(x,y) = x^4y - y^3 \] over the square \( D = \{(x,y); -1 \leq x \leq 1, -1 \leq y \leq 1 \} \).
[12] Use Lagrange multipliers to find the minimum and the maximum values of the function $f(x, y) = xy$ subject to the constraint $x^2 + y^2 = 4$. 
9. Note: questions (a) and (b) are mutually independent.

(a) Evaluate the iterated integral \( \int_0^1 \int_{y^2}^{2y} xe^y \, dx \, dy \). [Hint: you might want to reverse the order of integration.]

(b) Sketch the solid bounded by the plane \( z = 2y \), the \( xy \)-plane and the parabolic cylinders \( y = x^2 \) and \( y = 2 - x^2 \), then compute its volume using double integrals.
10. Figure below shows the graph of the curve $r^2 = 4 \cos 2\theta$ (it is called the lemniscate). Use double integrals to find the area enclosed by one leaf of this curve.