MATH 2720 Multivariable Calculus
TEST 2
March 11, 2009
(5:30-6:30, 205 Armes)

NAME: ___________________  Student number: ______

(If you need more space use the backside and indicate that you have done so.)

[8]  1. Consider the function \( z = x^3(1 + y^2) \).
(a) Find the slope of the line passing through the point (1,1,2) and tangent to the
curve of intersection of the surface \( z = x^3(1 + y^2) \) and the plane \( y = 1 \).
(b) Evaluate \( \frac{\partial^2 z}{\partial x \partial y} \) and \( \frac{\partial^2 z}{\partial y \partial x} \).

[7]  2. A weather balloon moves along the curve \( x = t, y = 2t, z = t - t^3 \), where \( t \)
stands for the elapsed time measured in hours (and \( x, y \) and \( z \) are the coordinates of the
balloon). The thermometer attached to the balloon gives the temperature of
\( T(x,y,z,t) = \frac{-xy}{1 + x} \) (in degrees Celsius). Find the rate of change of the temperature
at the time when \( t = 1 \). [Hint: draw a tree-diagram and use the Chain rule.]
3. Consider the function \( f(x,y) = y^2e^x \).
(a) Find the directional derivative of this function at the point \( P(0,1) \) and in the direction of the unit vector \( u = \left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right) \).
(b) Find the unit vector in the direction in which \( f \) increases most rapidly at \( P \) and give the rate of change in that direction. Find the unit vector in the direction in which \( f \) decreases most rapidly at \( P \) and give the rate of change in that direction.

4. Find the equation of the tangent plane to the surface defined by \( xy + yz + xz = 11 \) at the point \( (1,2,3) \).
5. Find the point in the plane \( x - y + z = 1 \) that is closest to the point \((-1,1,2)\).
Justify your answer by using the second (partial) derivative test.