MATH 2720 Multivariable Calculus
TEST 2
March 11, 2009
(5:30-6:30, 205 Armes)

NAME: ________________ **Student number:** ______

(If you need more space use the backside and indicate that you have done so.)

[8] **1.** Consider the function $z = x^2(1+y^2)$.

(a) Find the slope of the line passing through the point (1,1,2) and tangent to the curve of intersection of the surface $z = x^2(1+y^2)$ and the plane $y=1$.

(b) Evaluate $\frac{\partial^2 z}{\partial x \partial x}$ and $\frac{\partial^2 z}{\partial y \partial x}$.

[7] **2.** A weather balloon moves along the curve $x=t$, $y=2t$, $z=t-t^2$, where t stands for the elapsed time measured in hours (and x, y and z are the coordinates of the balloon). The thermometer attached to the balloon gives the temperature of $T(x,y,z,t) = \frac{xy}{1+z}(1+t)$ (in degrees Celsius). Find the rate of change of the temperature at the time when $t=1$. [Hint: draw a tree-diagram and use the Chain rule.]

[9] 3. Consider the function $f(x,y) = y^2e^{2x}$.

(a) Find the directional derivative of this function at the point $P(0,1)$ and in the direction of the unit vector $\mathbf{u} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

(b) Find the unit vector in the direction in which f increases most rapidly at P and give the rate of change in that direction. Find the unit vector in the direction in which f decreases most rapidly at P and give the rate of change in that direction.

[5] 4. Find the equation of the tangent plane to the surface defined by $xy + yz + xz = 11$ at the point $(1,2,3)$.

[11] 5. Find the point in the plane $x - y + z = 1$ that is closest to the point $(-1,1,2)$. Justify your answer by using the second (partial) derivative test.