# THE UNIVERSITY OF MANITOBA

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| 9 a.m., 2007 12 12 | Final Examination |
| **Paper number** 325 | **Page** 1 **of** 7 |
| **Course** MATH 3120 | **Time** 2 hours |
| **Examination** Applied Discrete Mathematics | **Examiner** R. S. D. Thomas |

Cover sheet

Instructions

Answer all questions on the examination paper. If you do not have enough room, use the back of the *previous* page. No aids.

Name (printed) ______________________

Student Number ______________

Seat Number __________

Signature ______________________

| Marks | |
|---|---|
| 1 | /5 |
| 2 | /12 |
| 3 | /5 |
| 4 | /6 |
| 5 | /8 |
| 6 | /4 |
| 7 | /7 |
| 8 | /11 |
| Total | /58 |

Some identities of set theory, logic, and Boolean algebra in logic notation

1. $P \vee \bot \Leftrightarrow P$
2. $P \wedge \top \Leftrightarrow P$
3. $P \vee Q \Leftrightarrow Q \vee P$
4. $P \wedge Q \Leftrightarrow Q \wedge P$
5. $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$
6. $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$
7. $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$
8. $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$
9. $P \vee \neg P \Leftrightarrow \top$
10. $P \wedge \neg P \Leftrightarrow \bot$
11. $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$
12. $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$
13. $P \vee P \Leftrightarrow P$
14. $P \wedge P \Leftrightarrow P$
15. $P \vee \top \Leftrightarrow \top$
16. $P \wedge \bot \Leftrightarrow \bot$
17. $P \wedge (P \vee Q) \Leftrightarrow P$
18. $P \vee (P \wedge Q) \Leftrightarrow P$
19. $\neg(\neg P) \Leftrightarrow P$

Question Number 1 — Value 5 out of 58

(a) State the theorem of Lagrange for groups. What does it talk about, and what does it say about them?

(b) What does the theorem of Lagrange tell you about the symmetry group of rotations of a regular pentagon around its centre?

Question Number 2 — Value 12 out of 58

(a) The recursive form of the algorithm EUCLID calls itself as well as being called. If it were called with the inputs 19 and 23, what calls altogether would be made before the solution was found? State each pair of inputs.

(b) Find the greatest common divisor of 19 and 23 using the algorithm EUCLID.

(c) Using the result of (b), find the multiplicative inverse of 19 in the ring $Z_{23}$.

Question Number 3 Value 5 out of 58

Consider the set of propositions, including $\top$ and $\bot$, together with the operation $\vee$ of the identities on page 1. Check it against the group definition to show which of the features of the definition are satisfied and which are not satisfied. The result is a proof that the system is a group or that it is not a group. (Identity 3 means that, if it is a group, it is an abelian group. This is not part of what you need to check.)

Question Number 4 Value 6 out of 58

(a) Translate the logic identity # 17 on page 1 into both set-theoretic and Boolean-algebra notations.

(b) Prove the logic identity # 17 in logical notation by the method of truth tables.

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Question Number 5 — Value 8 out of 58

Prove by induction (no marks for other methods) on $m$—not $n$—the combinatorial identity for general $n > 0$ and $m = 1, 2, \ldots$,

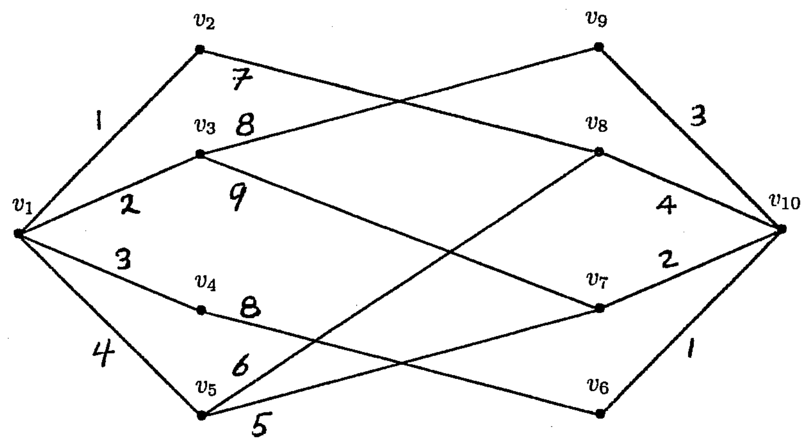
$$\binom{m+n}{n} = \binom{n-1}{n-1} + \binom{n}{n-1} + \ldots + \binom{m+n-1}{n-1}.$$

Question Number 6 Value 4 out of 58

Prove, directly or indirectly, that if a connected graph with $n$ vertices has more than $n-1$ edges then it has a cycle. You may use any graph-theory facts that you know and that are not equivalent to what you are to prove.

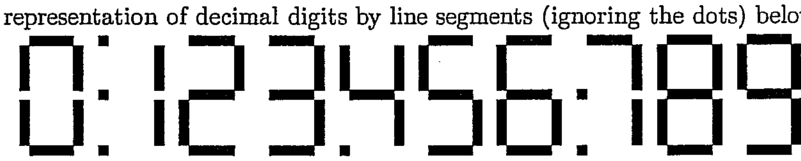
Question Number 7 Value 7 out of 58

Use the algorithm Maurer and Ralston call DIJKSTRA, modified as necessary, to determine the shortest path from the left side of the graph below to the right side, assuming that all edges are directed from left to right.



Question Number 8 | Value 11 out of 58

In the representation of decimal digits by line segments (ignoring the dots) below:



one can wonder whether it is simplest to insert the top segment in the 6 or the bottom segment in the 9 or both or neither, whatever one may think of the appearance. Both digits are perfectly recognizable with or without those segments. For input representing decimal digits by four bits $(w, x, y, z)$, decide whether the representations by boolean functions of the two segments at issue are simpler with or without their being used for 6 and for 9. Use the method of Karnaugh maps representing lighting up by 1, not lighting up by 0, 6 in one and 9 in another by ?, and don't-care places by $\times$. Write down expressions for what you regard as the simpler choice for the value of ? in each case, and give your decision whether inclusion or exclusion of each is the simpler option.

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