MATH 3120 Applied Discrete Mathematics Test 1 2007 10 17

Write on supplied long paper and hand in at the end of the hour with your name and student number. Total of marks, indicated in brackets, is 30.

[7] 1. Prove, using mathematical induction, that for every positive integer \( n \)

\[
\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}.
\]

2. [4] (a) Describe in a few words precisely what the recursive procedure TOH?? of p. 114, question 13, accomplishes and, physically speaking, how it does so when it acts on a lawful configuration of enough disks to do what it wants to do and \( s \neq r \). Do not use the rules of the TOH game as guides. And do not confuse variable assignment \( 'a \leftarrow b' \) with the authors' operation \( '\text{robot}(a \rightarrow b)' \). The procedure is described as follows:

```plaintext
procedure TOH??(in n, r, s)
robot(\( r \rightarrow b - r - s \))
if \( n > 1 \) then TOH??(n - 1, r, s) endif
robot(\( b - r - s \rightarrow s \))
endpro
```

(b) Write an iterative procedure TOH??-It to do the same thing (no marks if it does something else). Your answer to (b) can help you to improve your answer to (a), since in (a) you should be equally describing the physical results of both procedures.

3. [4] (a) An example of a graphical representation of a relation on a finite set was discussed in lectures: \( V = \{ \pm 1, \pm i \} \) with edge \( (a, b) \) if and only if \( a^2 = b \). Draw the more complicated digraph on the same vertices with an edge \( (a, b) \) representing 'for some positive integer \( n \), \( a^n = b' \). Note the quantifier. (It does not mean that 'for some one particular value of \( n \) \( a^n = b' \) but 'there exists a positive integer value of \( n \) such that \( a^n = b' \).

(b) Name one of the conditions for the relation to be an equivalence that fails for this relation or show that it is an equivalence. Indicate the evidence for the failure of the condition that you name or for the relation's being an equivalence.

[7] 4. Use the algorithm Maurer and Ralston call WARSHALL to determine the path matrix for the digraph below. Display intermediate results for marks.

```
\begin{itemize}
  \item \( v_1 \)
  \item \( v_2 \)
  \item \( v_3 \)
  \item \( v_4 \)
  \item \( v_5 \)
\end{itemize}
```