Midterm Examination #1 MATH3132 Mathematical Methods for Engineers 3

Time: 60 Minutes

Student Name (Print): ____________________________
Student Signature: _______________________________
Student Number: ________________________________

INSTRUCTIONS:
1. No aids permitted.
2. If insufficient space is provided for a solution to a problem, continue your work on the back of the previous page.
3. Check that your examination booklet contains 4 pages.

Formulae
\[ \oint_C P \, dx + Q \, dy = \iint_R \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA \]
\[ \iint_S F \cdot \hat{n} \, dS = \iiint_V \nabla \cdot F \, dV \]
\[ \oint_C F \cdot \hat{r} \, ds = \iint_S (\nabla \times F) \cdot \hat{n} \, dS \]

10 1. Evaluate the line integral
\[ \int_C 32y \, ds \]
where \( C \) is the curve \( x = y^2, \ z = 4 + x \) from the point \((4, 2, 8)\) to the point \((1, 1, 5)\).
2. Evaluate the line integral

\[ \int_C \frac{x}{x^2 + y^2} \, dx + \frac{y}{x^2 + y^2} \, dy, \]

where \( C \) consists of the three straight line segments joining successively the points \((2,0), (2,1), (-2,1), \) and \((-2,0).\)
3. Evaluate the surface integral

\[
\iint_S \left[ (x^2 + y^2)i + z^2 \right] \cdot \hat{n} dS,
\]

where \( S \) is the closed surface that bounds the volume enclosed by the surfaces

\[
x^2 + y^2 = 4, \quad z = 1, \quad z = -1,
\]

and \( \hat{n} \) is the unit inward pointing normal to \( S \).
4. Evaluate the surface integral
\[ \iint_S (y\mathbf{i} - x\mathbf{j} + z\mathbf{k}) \cdot \mathbf{n} \, dS \]
where \( S \) is that part of the surface \( z = 9 - x^2 - y^2 \) above the \( xy \)-plane, and \( \mathbf{n} \) is the unit normal to \( S \) with positive \( z \)-component.