INSTRUCTIONS TO STUDENTS:

This is a 60 minute exam. Please show your work clearly.

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 6 pages of questions, the last of which contains formulas. Please check that you have all the pages.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 40 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.
1. Evaluate the following line integral:

\[ \int_{C} 2x(1 + y + z) \, ds \]

where \( C \) is the curve \( x = 2t, y = t^2, z = t^2 \) for \( 0 \leq t \leq 1 \).
[3] 2. (a) Show that \[ \int_C \vec{F} \cdot d\vec{r} \] is independent of path where

\[ \vec{F} = (x^2 + yz) \hat{i} + (y^3 + zx) \hat{j} + (z^2 + xy) \hat{k} . \]

[4] 4. (b) Evaluate \[ \int_C \vec{F} \cdot d\vec{r} \] where \( \vec{F} \) is given in part (a) and C is the curve \( x = t^3 - t^2, \)
\( y = 1 - 3t^2 + t^3, \) \( z = (t - 1)^3, \) for \( 0 \leq t \leq 2. \)
3. Evaluate the closed line integral \[ \oint_C 3xy^2 \, dx + x^2y \, dy \] where \( C \) is the piecewise smooth curve that consists of the line segments joining (1, 3) to (4, 3); (4, 3) to (4, 1); (4, 1) to (1, 1) and (1, 1) to (1, 3).
4. Evaluate the surface integral \[ \iint_S z \, dS \] where \( S \) is the part of the plane \( z = x + y + 3 \) that lies inside the cylinder \( x^2 + y^2 = 1 \).
5. Evaluate the closed surface integral \( \iint_{\mathcal{S}} \vec{F} \cdot \hat{n} \, dS \) where

\[
\vec{F} = (x^3 + y \sin z)\hat{i} + (y^3 + z \sin x)\hat{j} + 3z\hat{k}
\]

and \( \hat{n} \) is the outward unit normal to the closed surface of the solid cylinder \( x^2 + y^2 \leq 4 \) between \( z = 1 \) and \( z = 3 \).
Some formulas:

Green's Theorem:
\[ \oint_C P \, dx + Q \, dy = \iint_A \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA \]

Divergence Theorem:
\[ \iiint_S \mathbf{F} \cdot n \, dS = \iiint_V \nabla \cdot \mathbf{F} \, dV \]