Math 3132 Midterm 1 (1 hr) Feb 12, 2010

To obtain full credit, intermediate steps in obtaining your answers must be given.

1. Show that the following line integral is independent of path and evaluate it

$$\int_C 2xydx + (x^2 + y)dy$$

with C as i) any path from $(0,1)$ to $(2,3)$, and ii) the path $\{(1 + 2\cos 2t, \sin 2t)\}$ starting with $t = 0$ and ending with $t = \pi$. (10 pts)

2. Evaluate the line integral

$$\oint_C (\sin x + 3y^2)dx + (2x - e^{-y^2})dy,$$

where C is the boundary curve of the finite region bounded by the parabola $y = 1 - x^2$ and the line $y = 0$ with clockwise orientation. (7 pts)

3. Verify the divergence theorem for the vector field $F = [x, 1, z\sqrt{x^2 + y^2}]$ in the cylinder $x^2 + y^2 = 1$ for $0 \leq z \leq 1$. (14 pts)

4. **Use Stokes theorem** to evaluate the line integral

$$\oint_C (2xy + y)dx + (x^2 + xy - 3y)dy + 2xzdz$$

where C is the curve given by the intersection of the surfaces $z = \sqrt{x^2 + y^2}$ and $z = 4$ and is oriented in the counterclockwise direction when viewed from the point $(0, 0, 10)$. Draw a diagram. (9 pts)