Math 3132 Midterm 1 (1 hr)  Feb 12, 2010
To obtain full credit, intermediate steps in obtaining your answers must be given.

1. Show that the following line integral is independent of path and evaluate it

\[ \int_C 2xydx + (x^2 + y)dy \]

with \( C \) as i) any path from \((0, 1)\) to \((2, 3)\), and ii) the path \( \{(1 + 2 \cos 2t, \sin 2t)\} \) starting with \( t = 0 \) and ending with \( t = \pi \). (10 pts)

2. Evaluate the line integral

\[ \oint_C (\sin x + 3y^2)dx + (2x - e^{-y^2})dy, \]

where \( C \) is the boundary curve of the finite region bounded by the parabola \( y = 1 - x^2 \) and the line \( y = 0 \) with clockwise orientation. (7 pts)

3. Verify the divergence theorem for the vector field \( F = [x, 1, z\sqrt{x^2 + y^2}] \) in the cylinder \( x^2 + y^2 = 1 \) for \( 0 \leq z \leq 1 \). (14 pts)

4. Use Stokes theorem to evaluate the line integral

\[ \oint_C (2xy + y)dx + (x^2 + xy - 3y)dy + 2xzdz \]

where \( C \) is the curve given by the intersection of the surfaces \( z = \sqrt{x^2 + y^2} \) and \( z = 4 \) and is oriented in the counterclockwise direction when viewed from the point \((0, 0, 10)\). Draw a diagram. (9 pts)